

doc. RNDr. Barbara Zitová, PhD

Digitální zpracování obrazu

DIZO



Důležité informace

Rozsah: LS, 2+2, Zk

<http://zoi.utia.cas.cz>


DIZO

CZECH UNIVERSITY Digital image processing : FNSPE-CTU in Prague

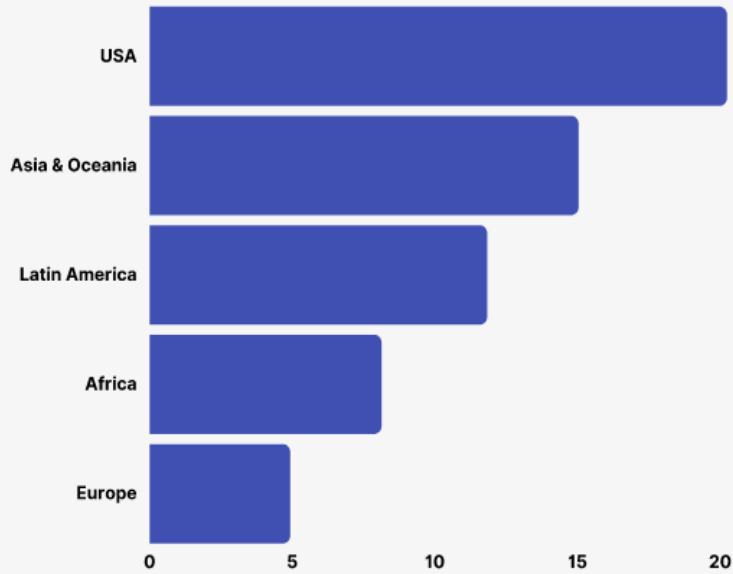
📌 COURSE DESCRIPTION

Foundational Training in Image processing and Pattern Recognition

This course emphasizes the conversion of images into digital form, initial processing techniques like noise reduction, improving contrast, and removing blur, as well as techniques for identifying edges, dividing images into segments, and applying geometric changes. Theoretical concepts will be

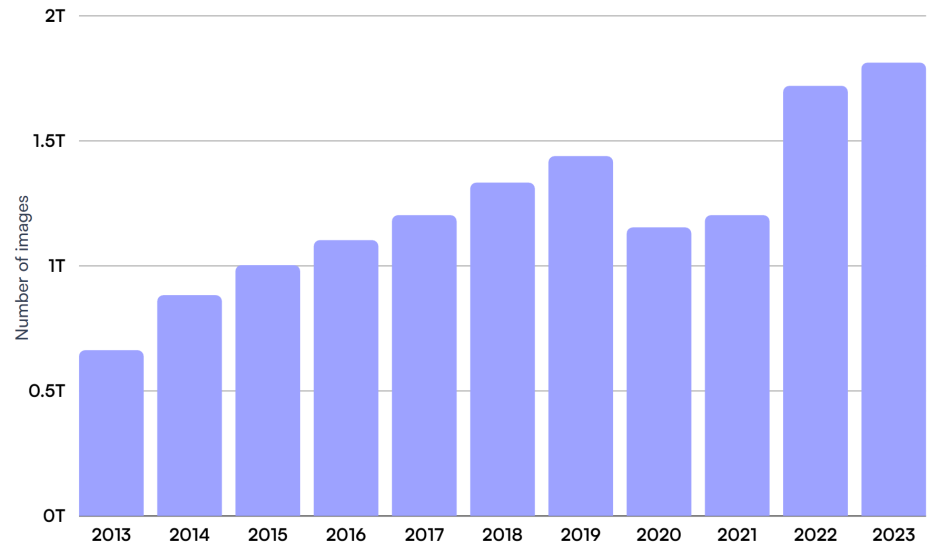


Photos taken per day



Number Of Photos Taken Each Year

(in trillions of photos)

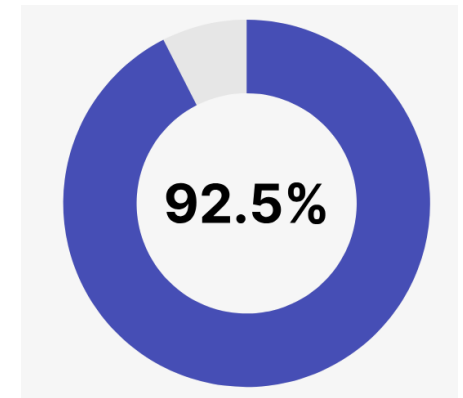


Source: Photutorial

Photutorial

1.81×10^{12} fotografií za rok $\sim 57,000$ za 1s

750×10^9 fotografií na Internetu
 $= \sim 6\%$



Navazující předměty

Speciální funkce a transformace ve zpracování obrazu

Variační metody ve zpracování obrazu

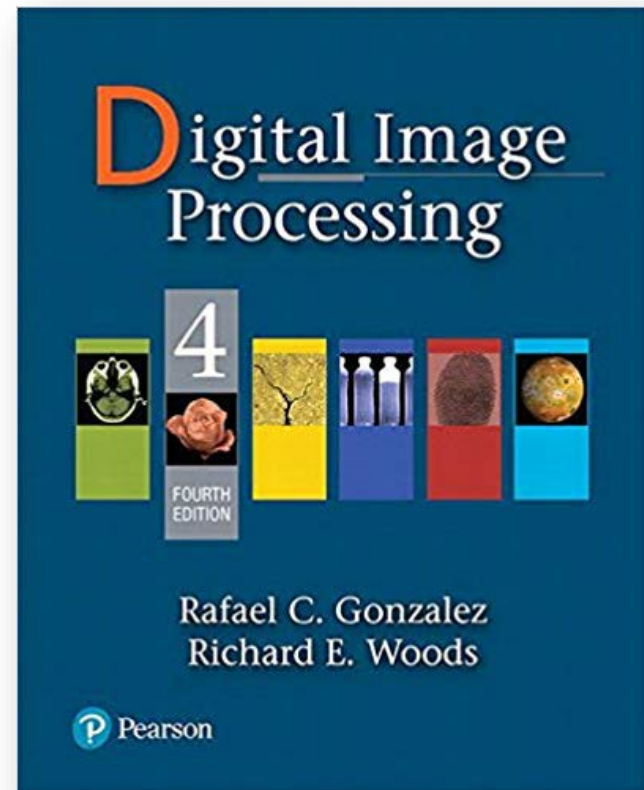
Strojové učení 1

Strojové učení 2

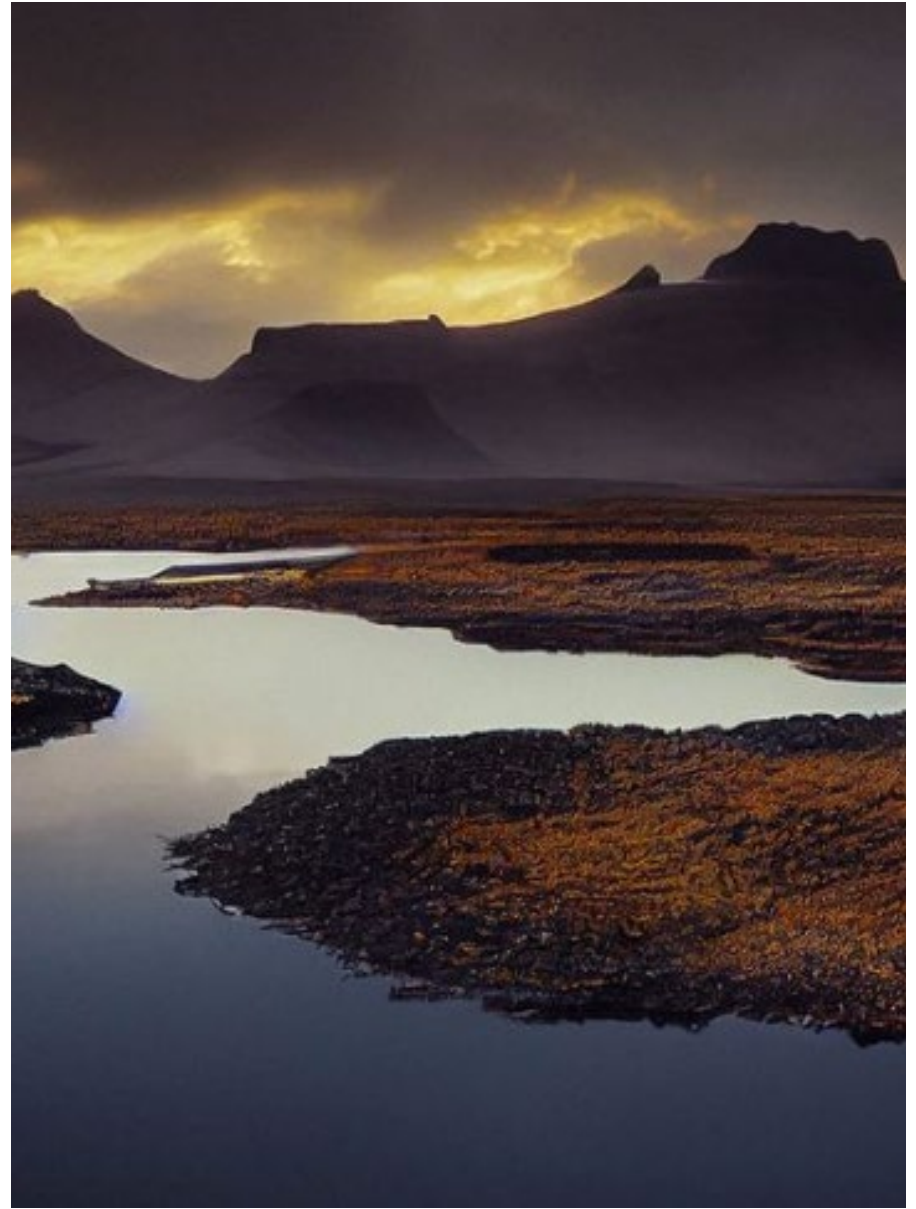
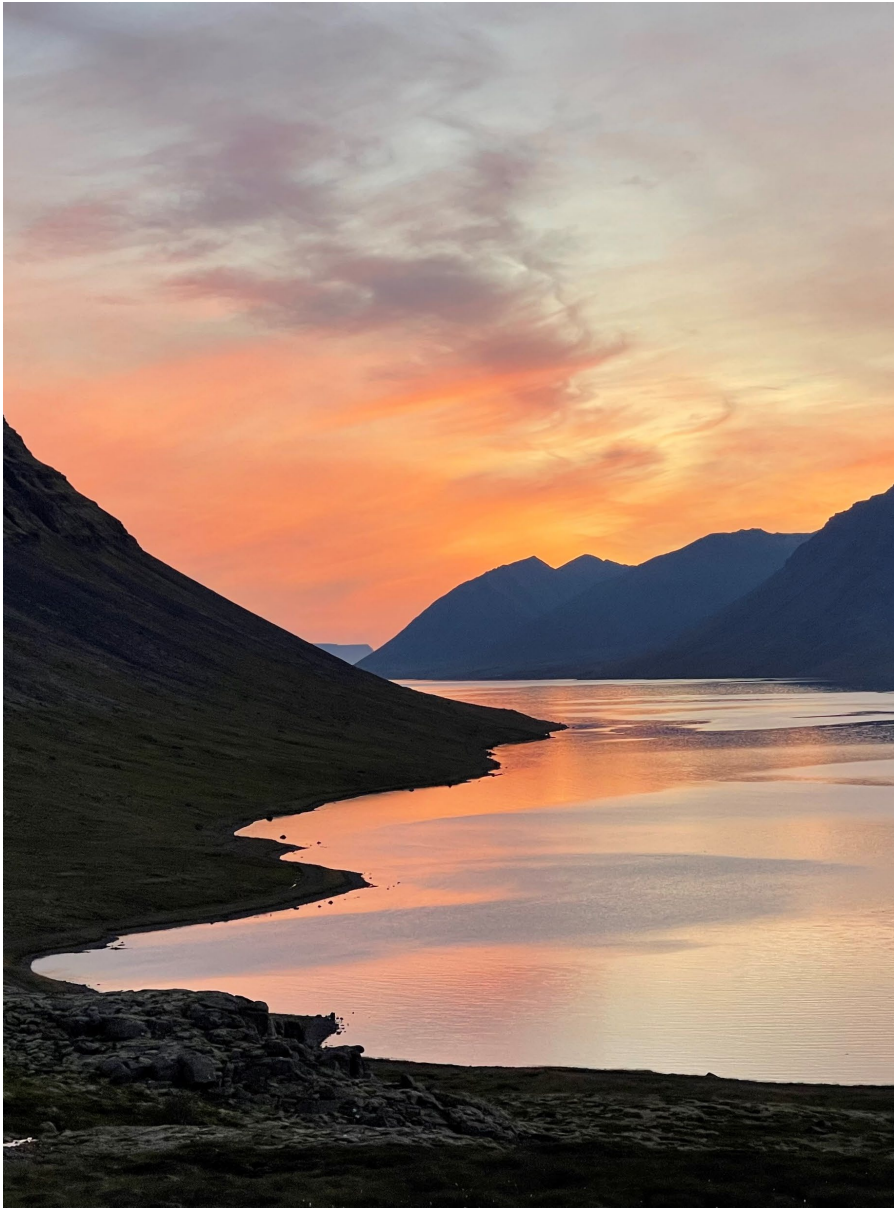
Digital Image Processing

4rd edition

Gonzalez & Woods





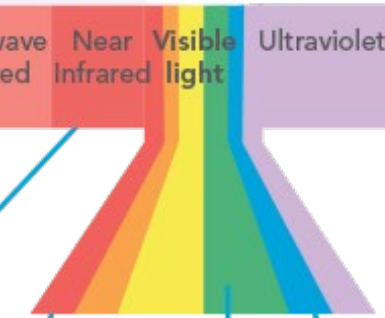




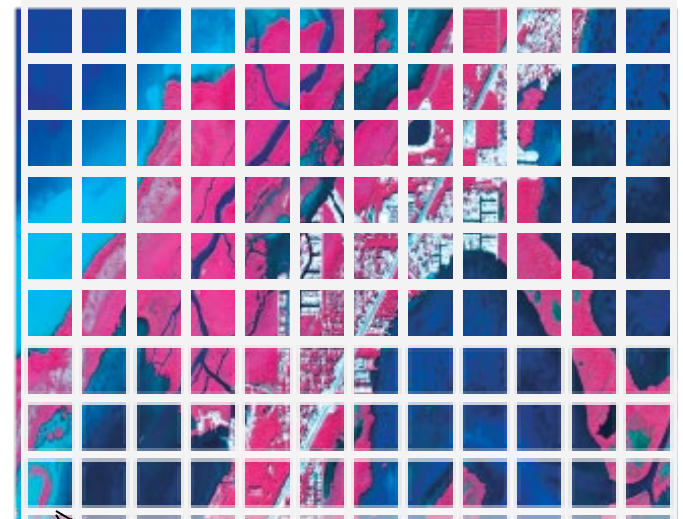
Avery Jos



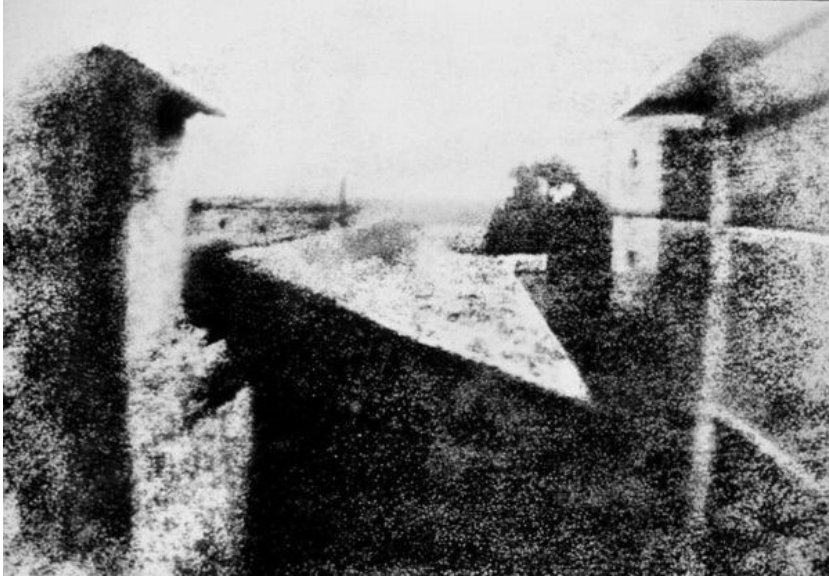




shortwave infrared band near infrared band red band green band blue band



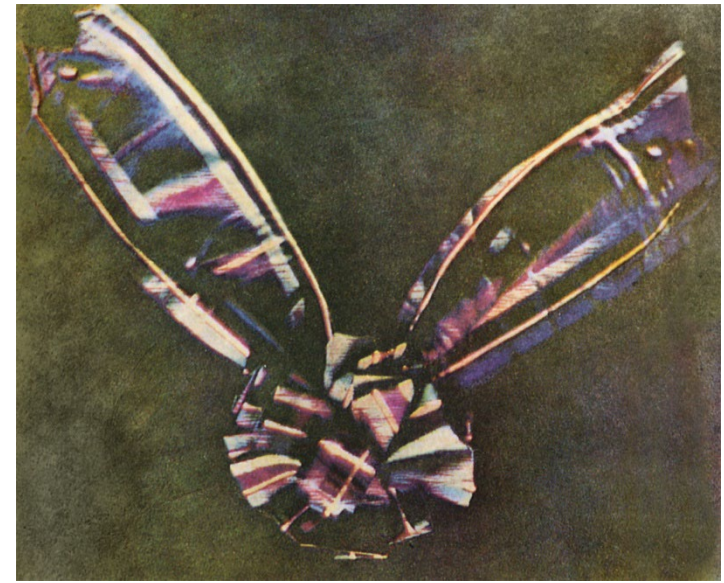
Historie



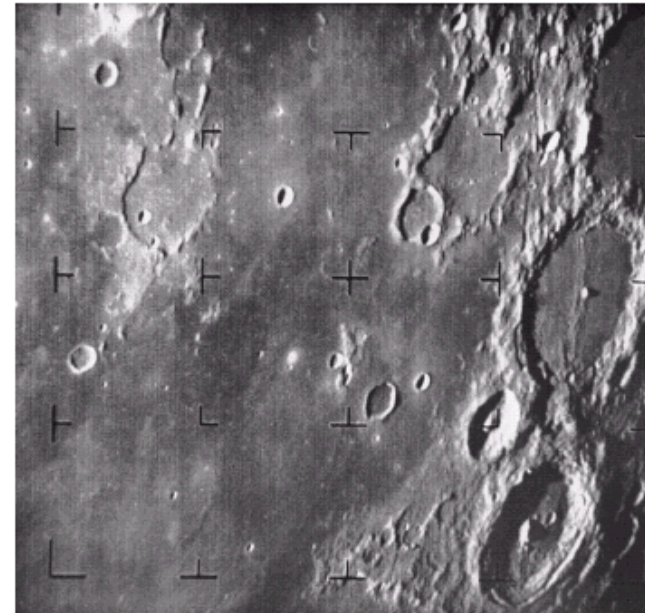
1827



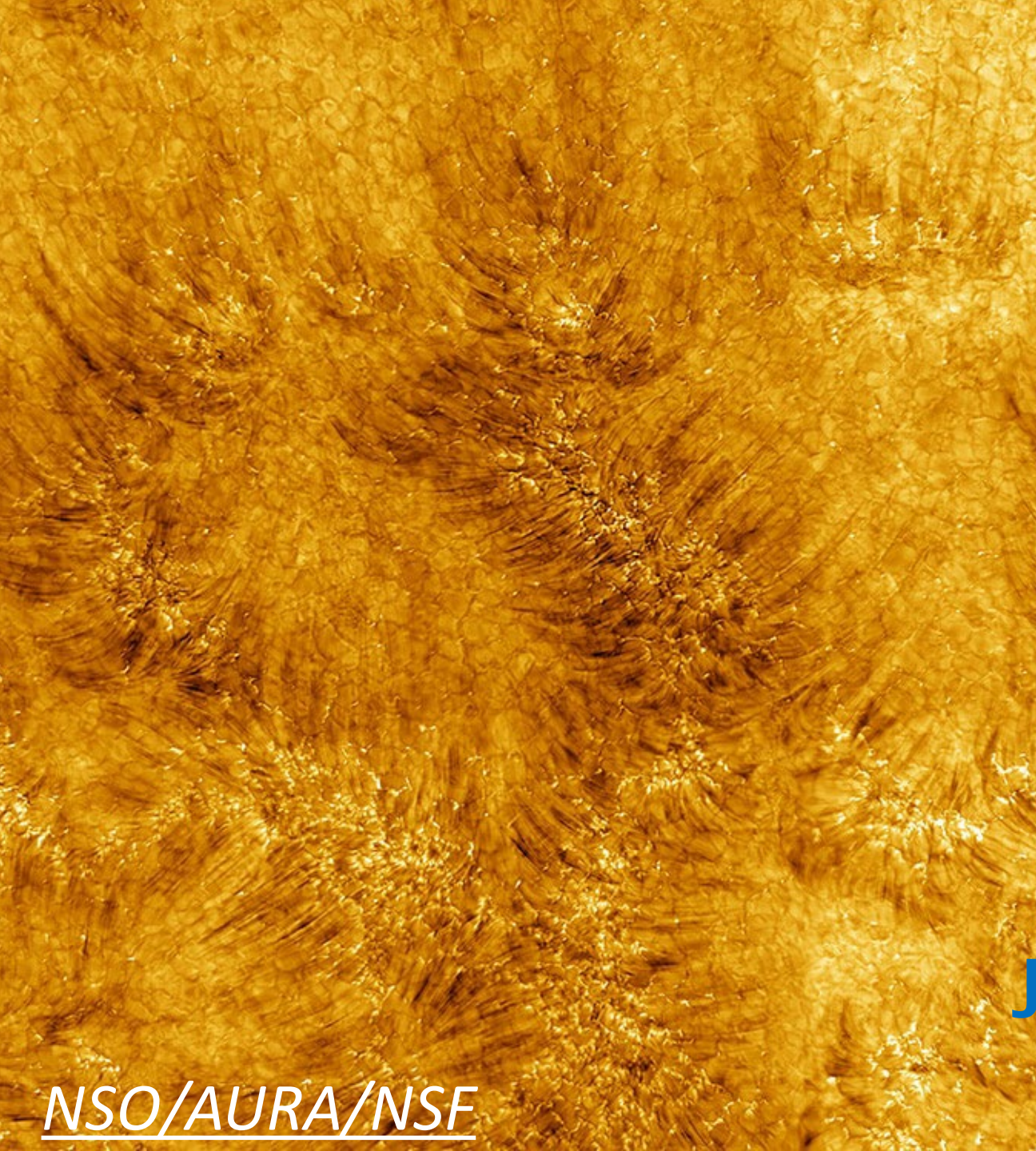
1921



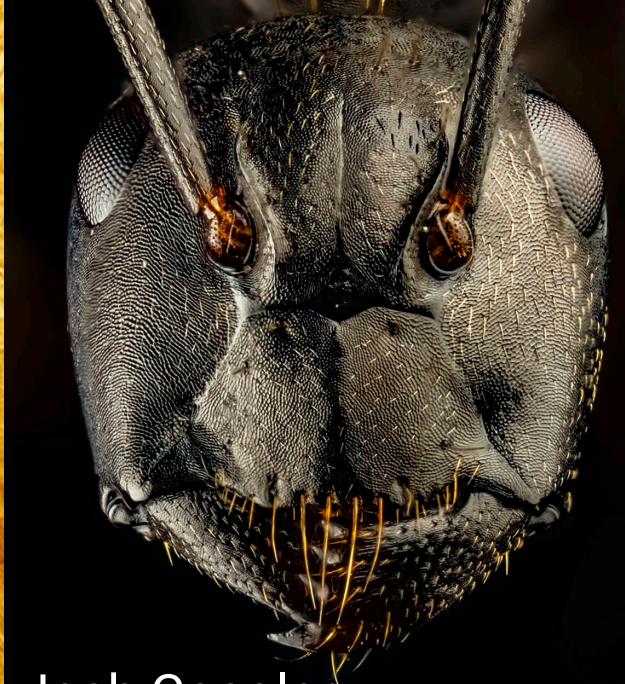
1861



1964



NSO/AURA/NSF

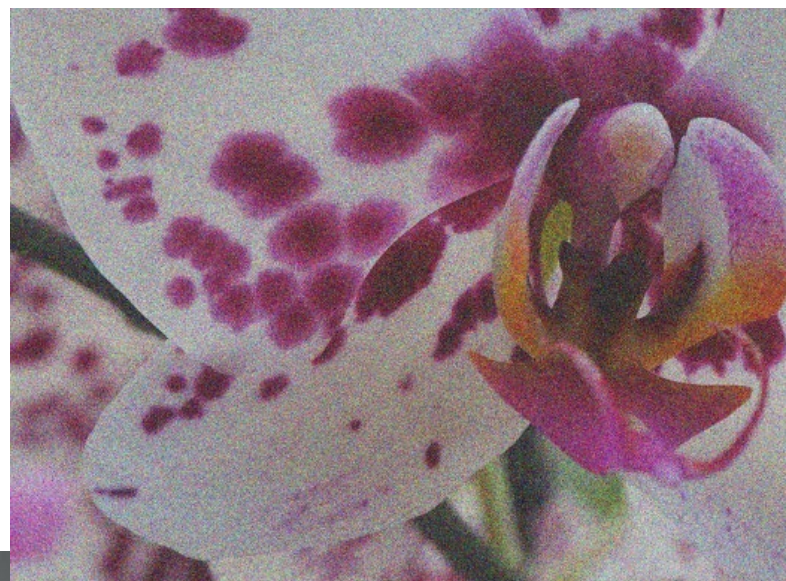


Josh Coogler



NASA

Fotografie nebývají ideální



Fotografie nebývají ideální



originál



výsledný snímek



geometrie

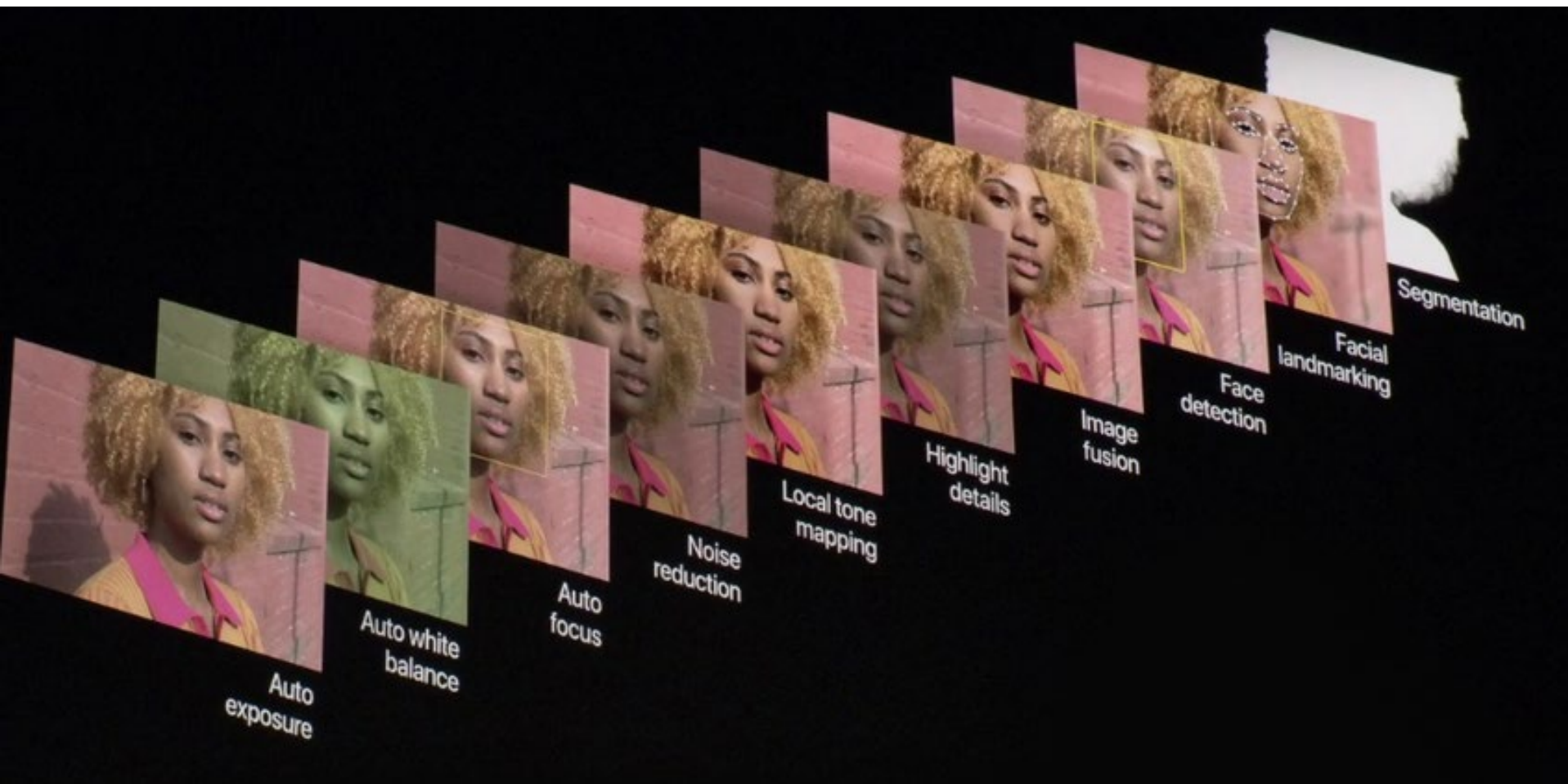


rozmazání

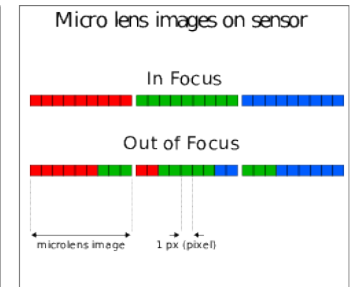
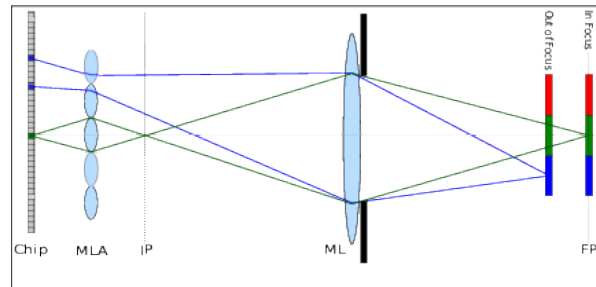


šum

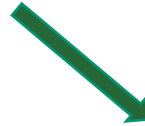
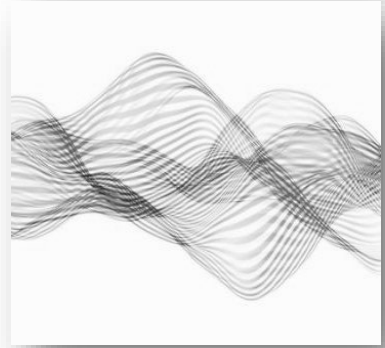
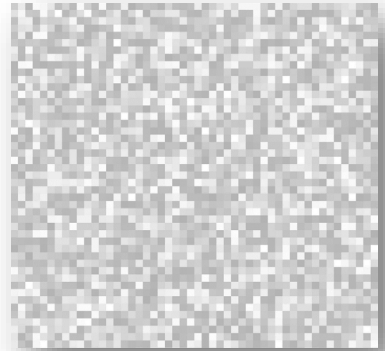
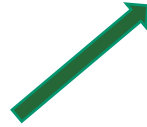


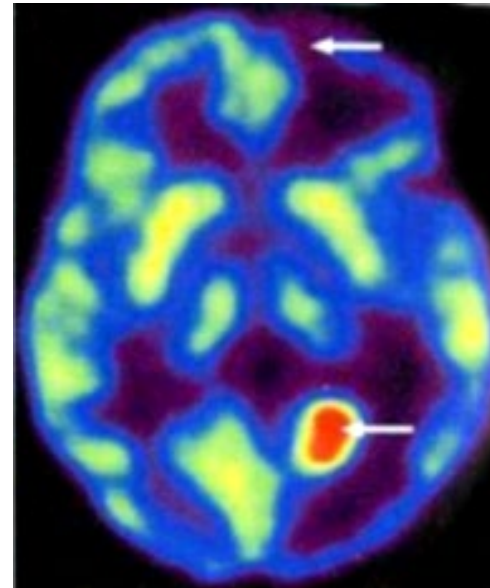
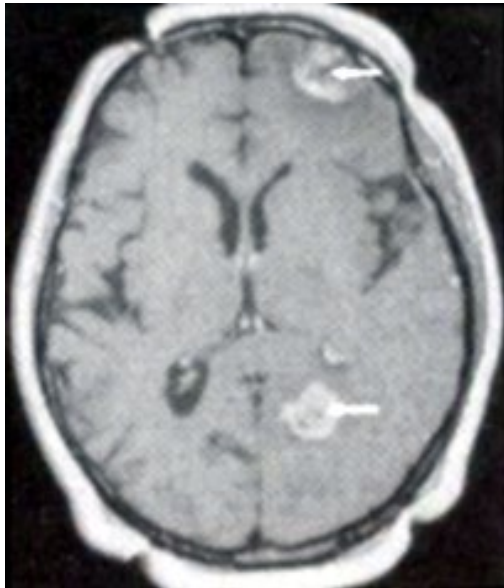


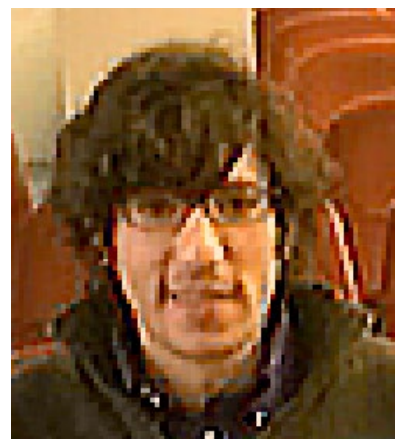
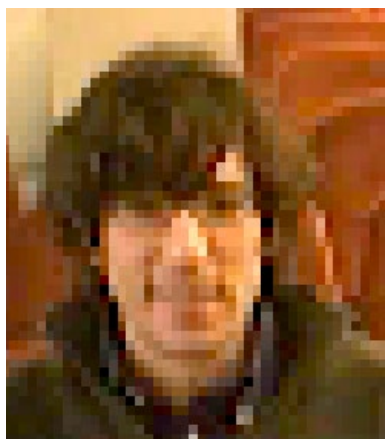
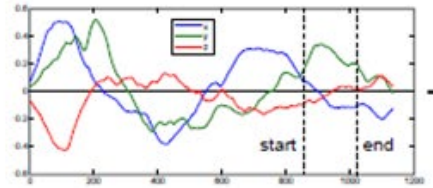
Light field camera







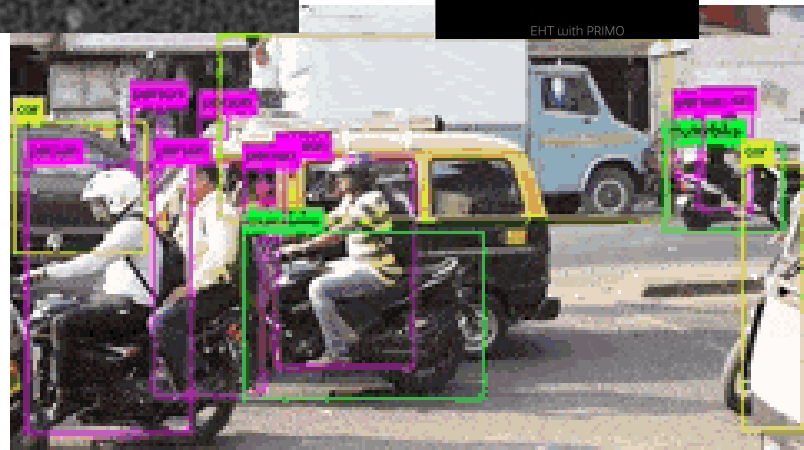
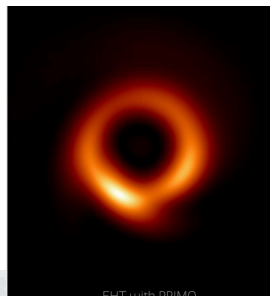
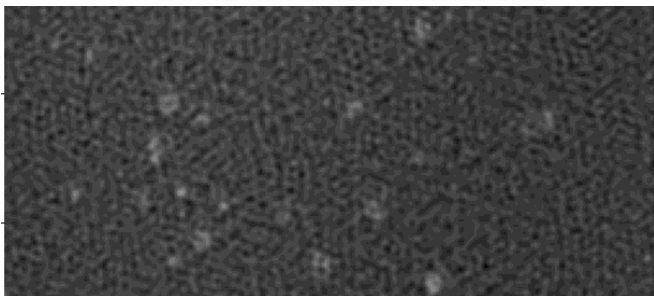
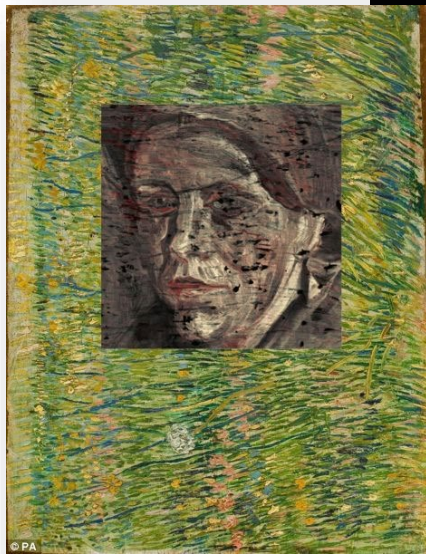
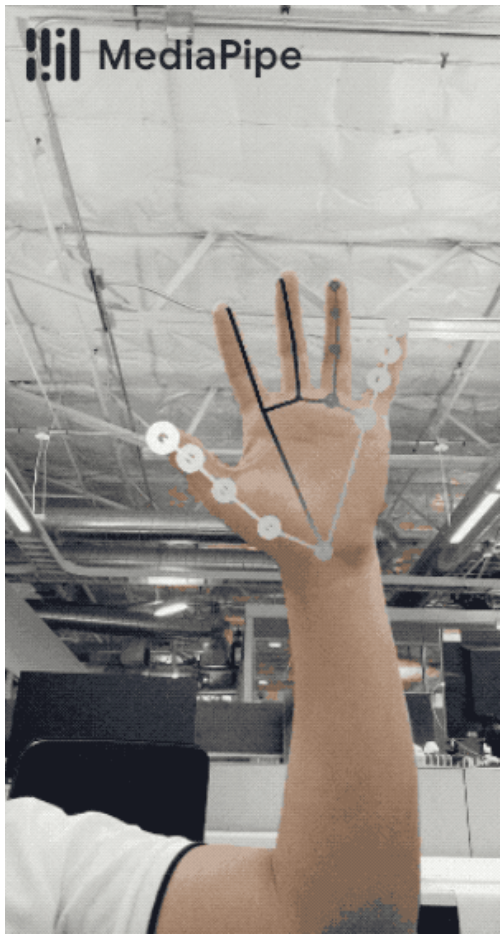


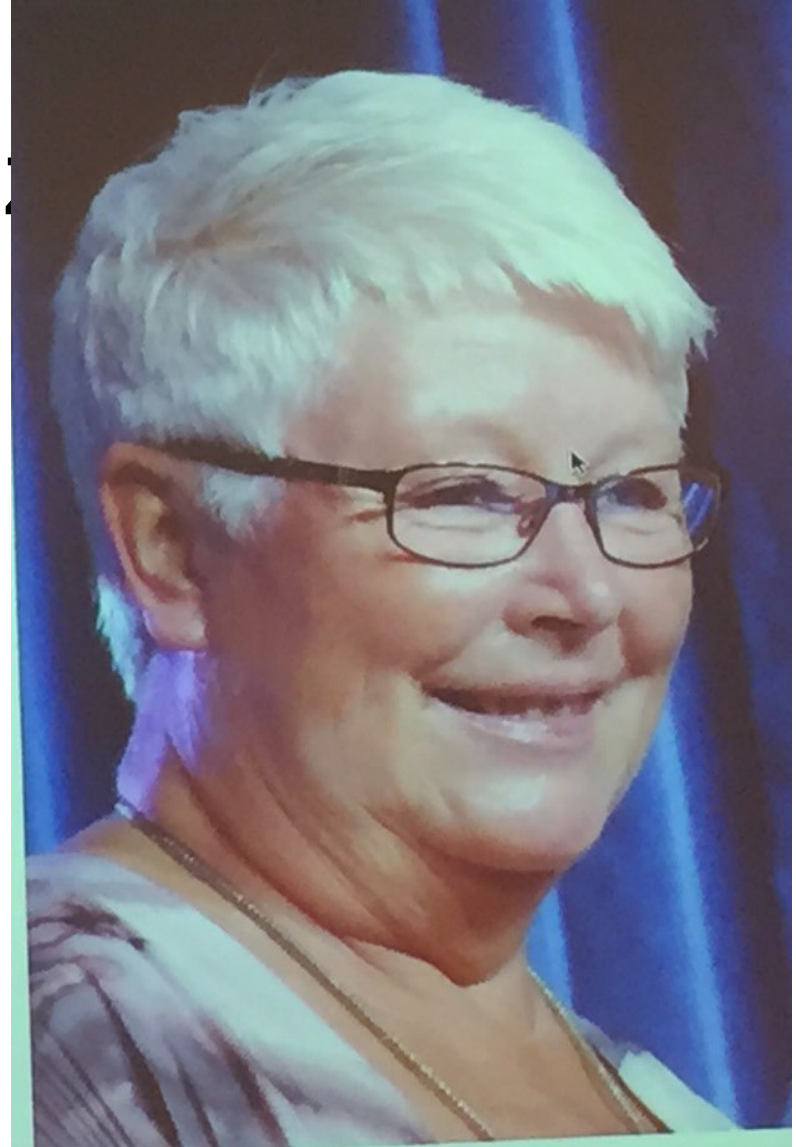




Hubble teleskop







Topics of DIZO

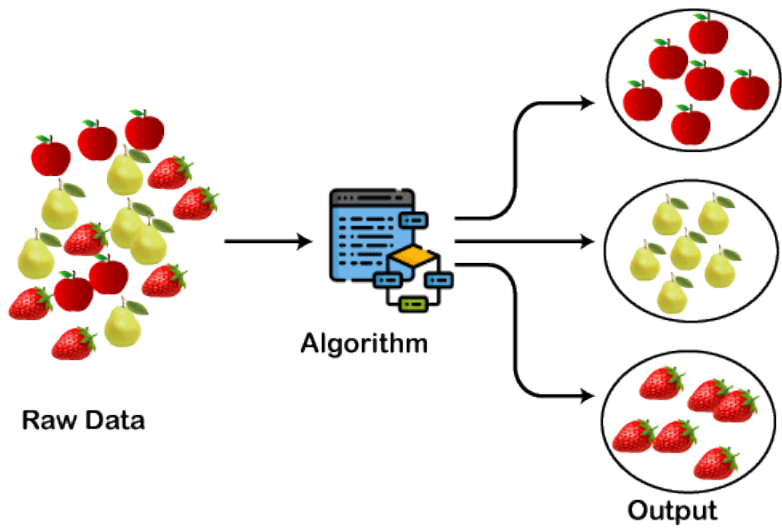
- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, ...)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)

Topics partially covered

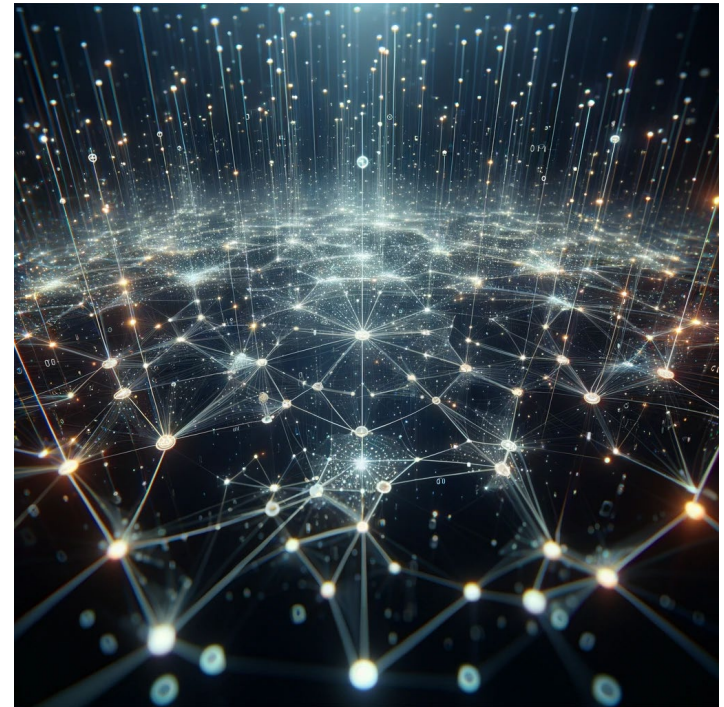
- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, ...)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)

Topics SFTO

- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, ...)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)



VS



velikost



barva

A

B

C

A

B

C

A

B

C



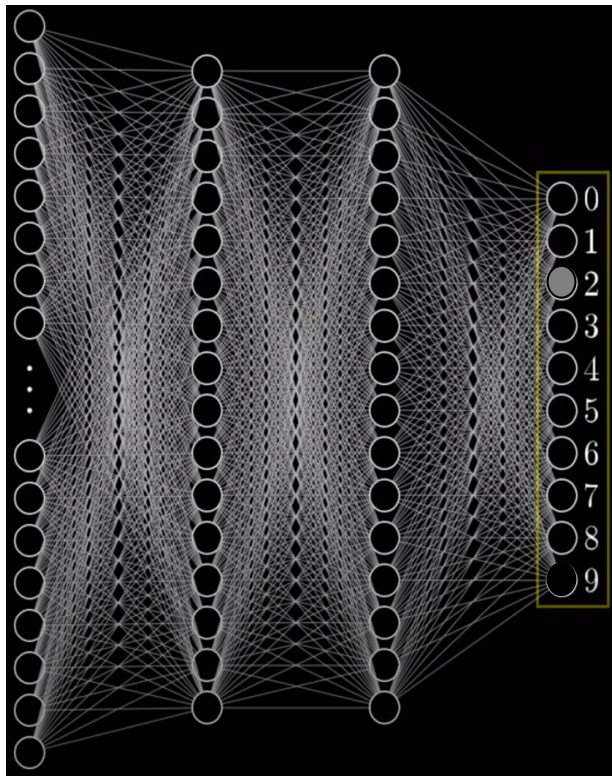
MELOUN



JABLKO

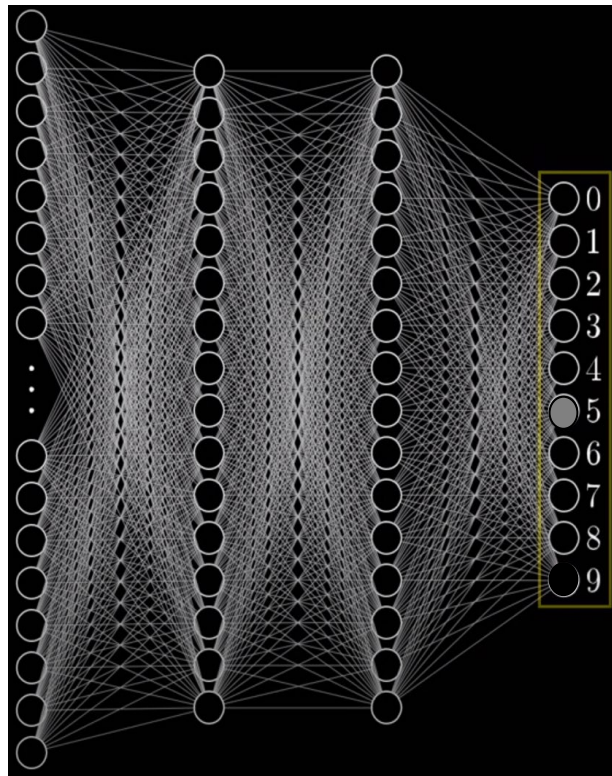






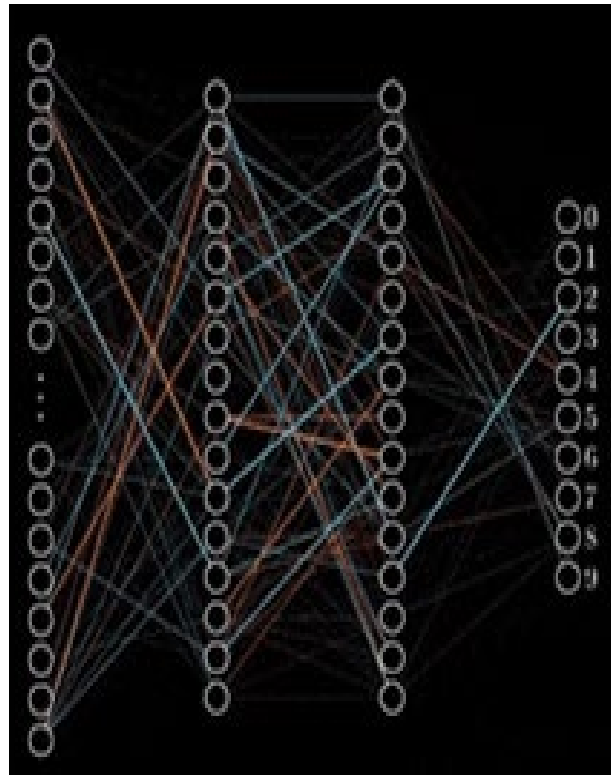
JABLKO



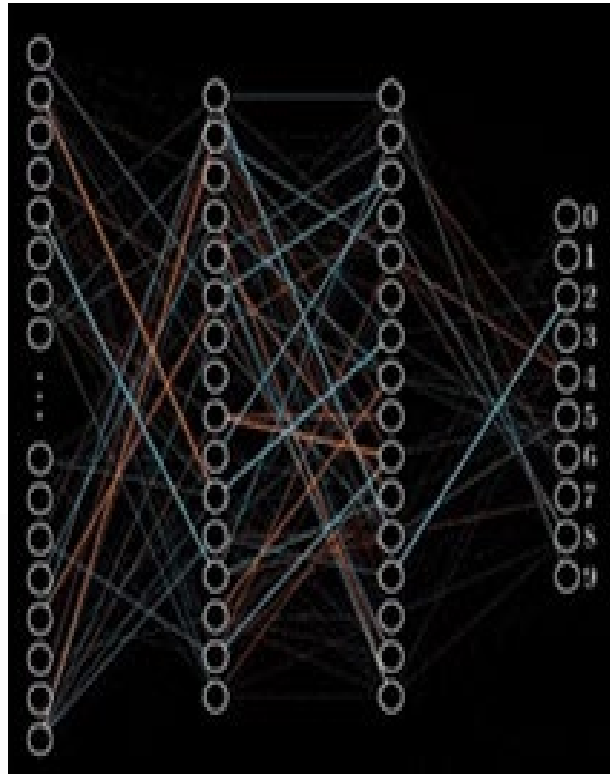


MELOUN



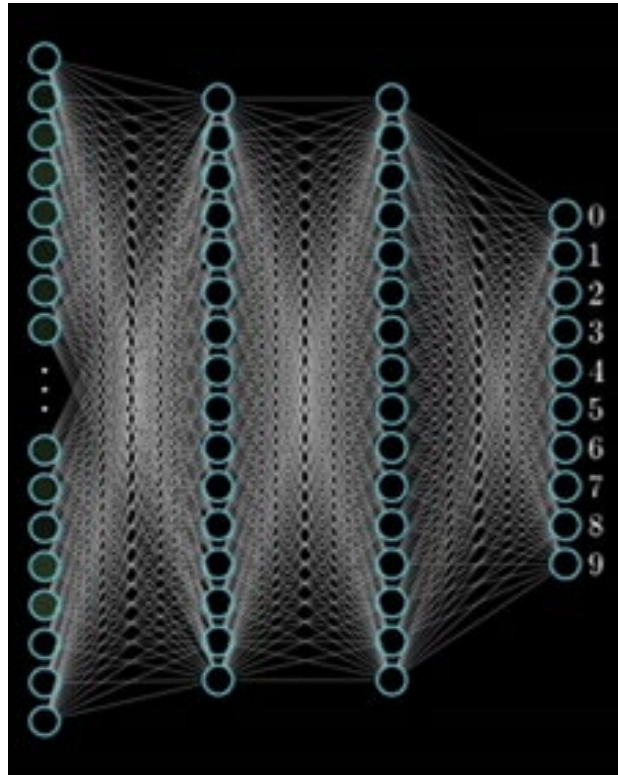


MELOUN



JABLKO





JABLKO





Matematické základy

- konvoluce
- Fourierova transformace

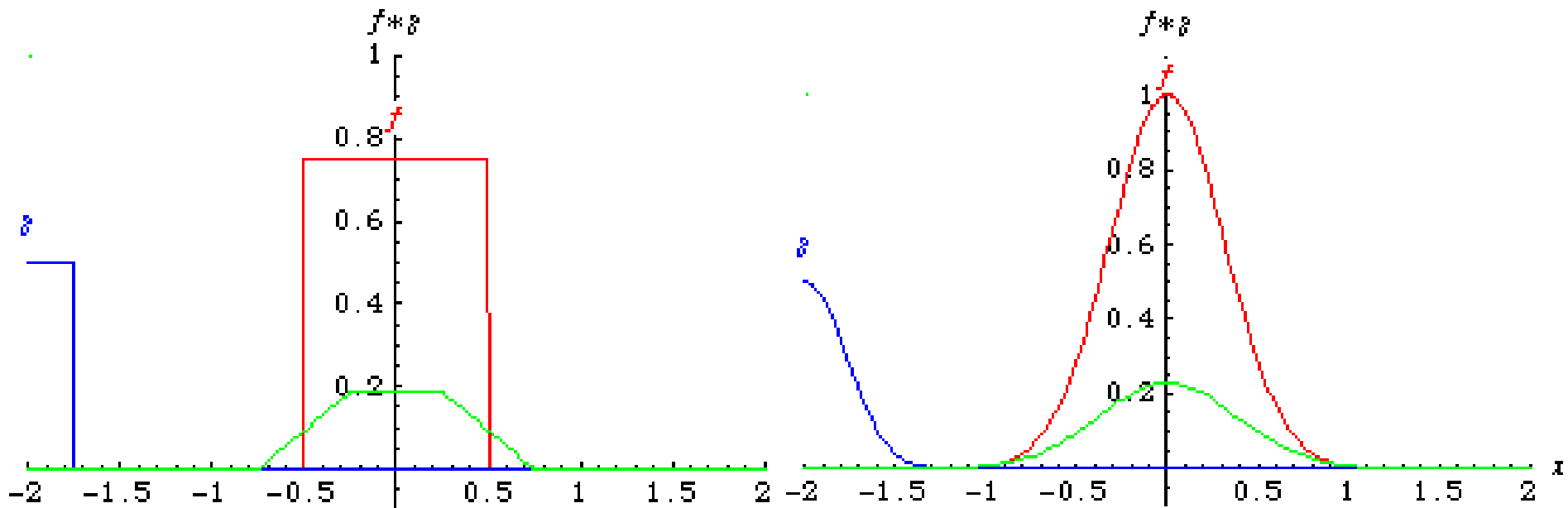
Konvoluce

- Jak moc se „překrývají“ dvě funkce
- definice ve spojitém případě
- vlastnosti
- delta funkce
- diskrétní konvoluce, okrajový jev

Konvoluce

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau$$

$$* : L_1 \times L_1 \rightarrow L_1$$



Vlastnosti

$$f * g = g * f$$

$$f * (g * h) = (f * g) * h$$

$$a(f * g) = (af) * g = f * (ag)$$

$$f * (g + h) = (f * g) + (f * h)$$

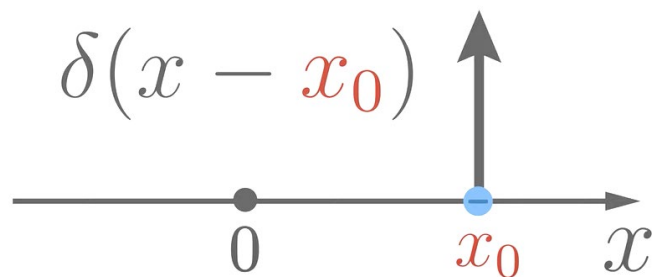
$$f * \delta = \delta * f = f$$

Dirac delta funkce

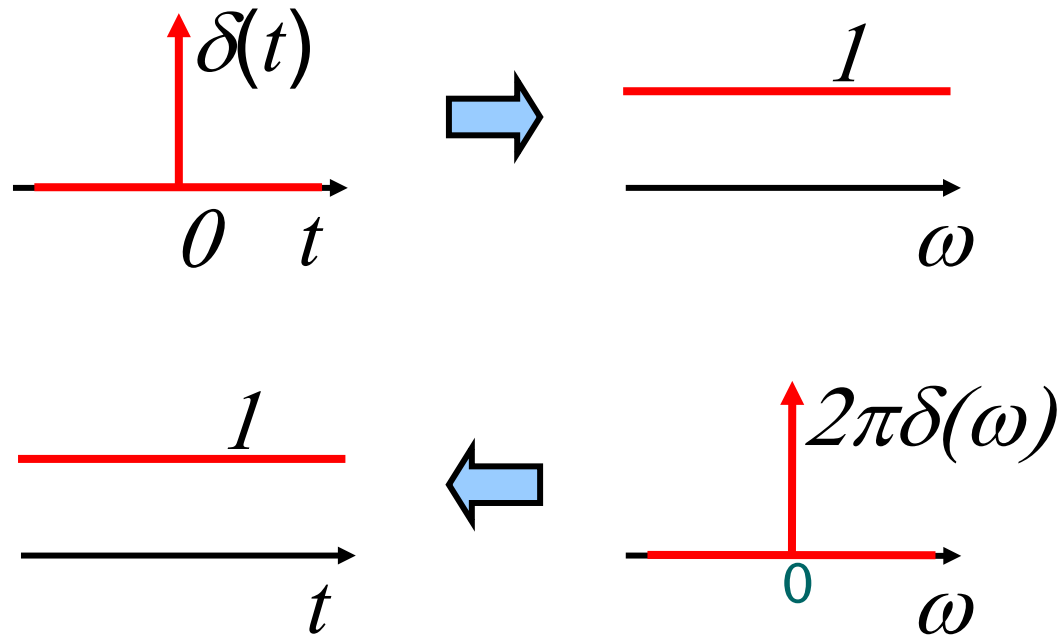
$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \quad \int \delta(x) dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\delta(x - a) dx = f(a)$$

$$\delta(ax) = \frac{\delta(x)}{|a|}$$



Dirac delta funkce



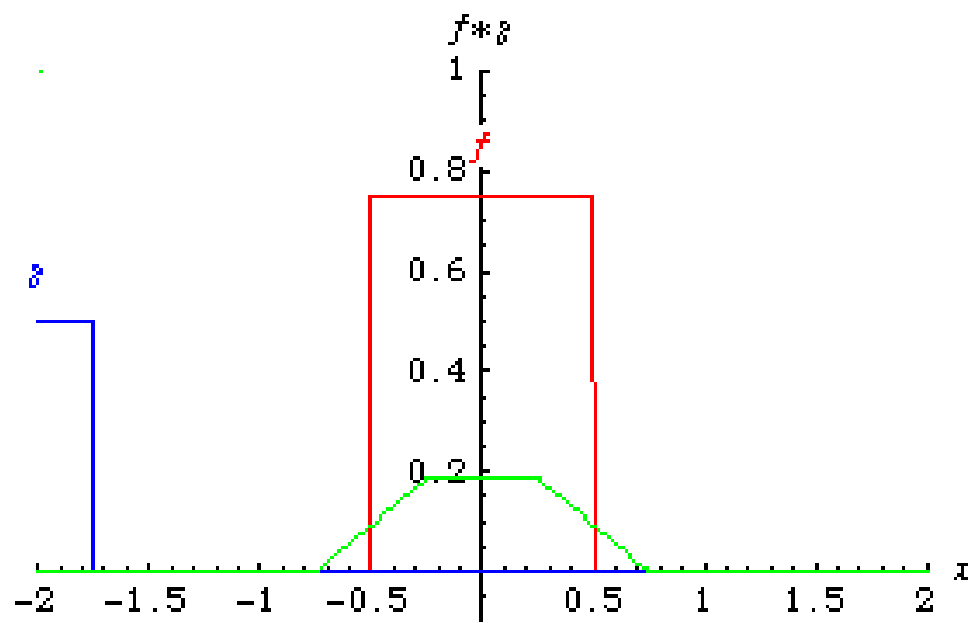
Konvoluce

- korelace

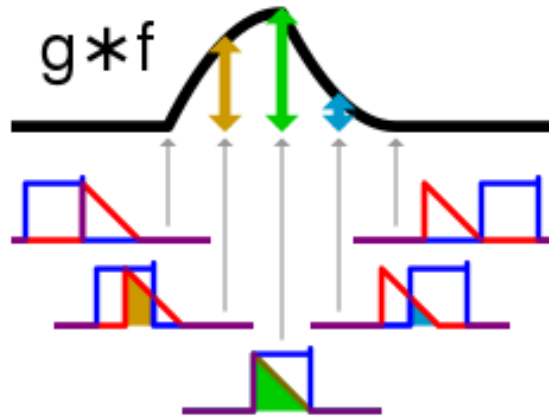
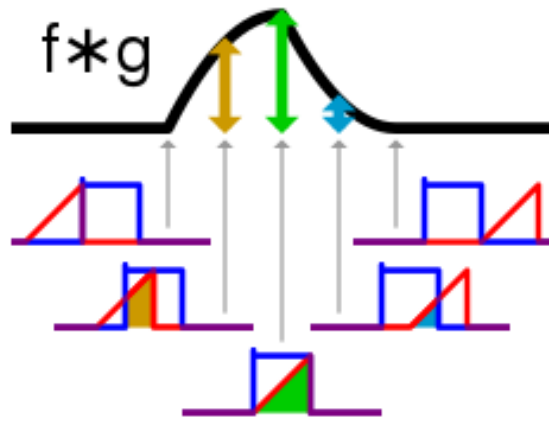
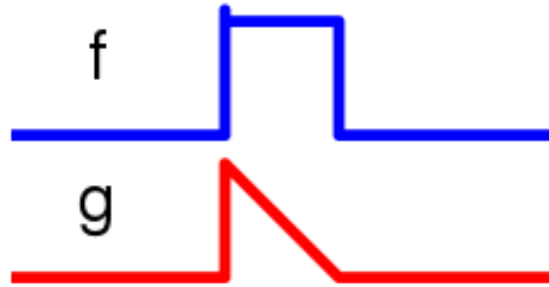
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

- konvoluce

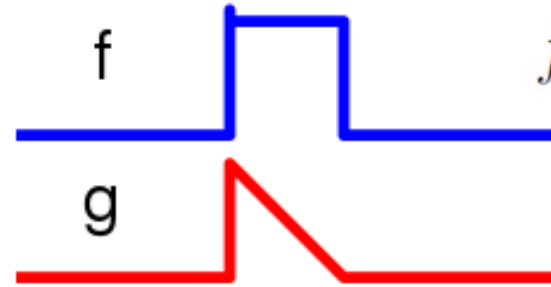
$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$



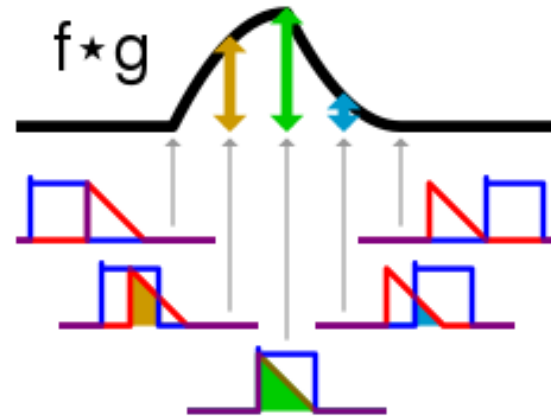
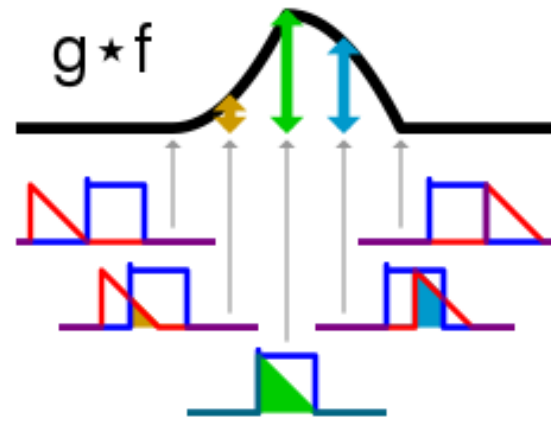
Convolution

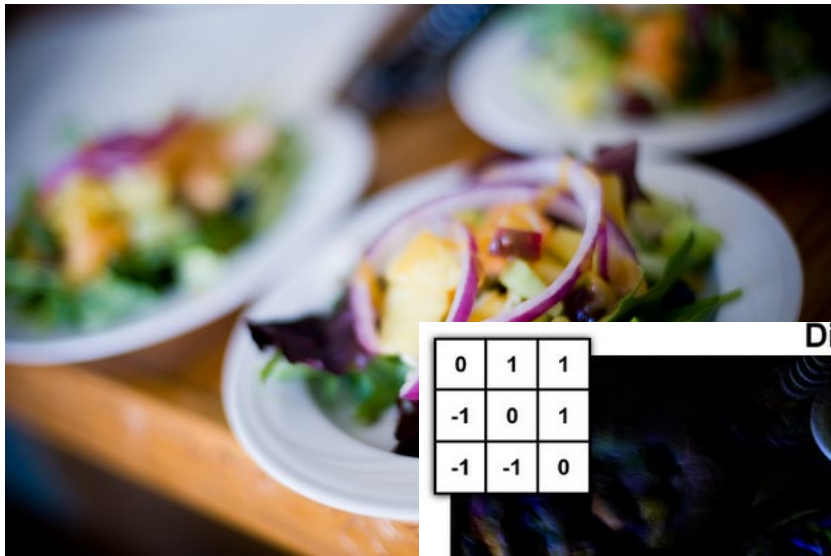


Cross-correlation



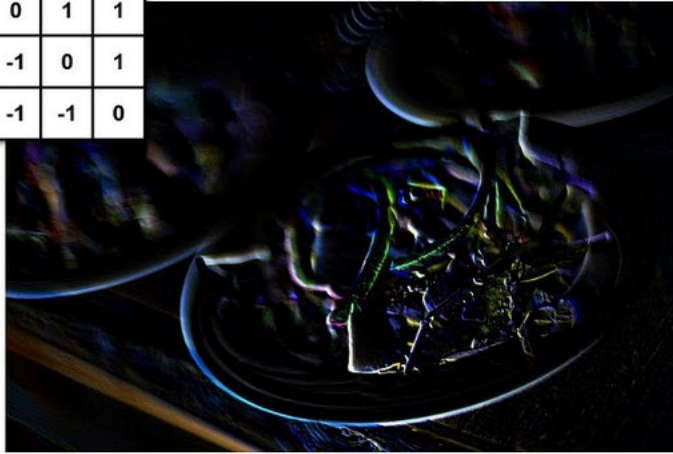
$$f \star g \neq g \star f$$





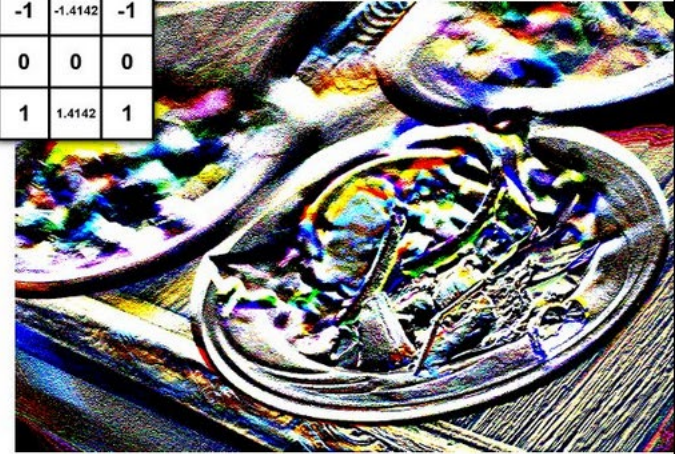
0	1	1
-1	0	1
-1	-1	0

Diagonal Prewitt



-1	-1.4142	-1
0	0	0
1	1.4142	1

Horizontal Frei-Chen



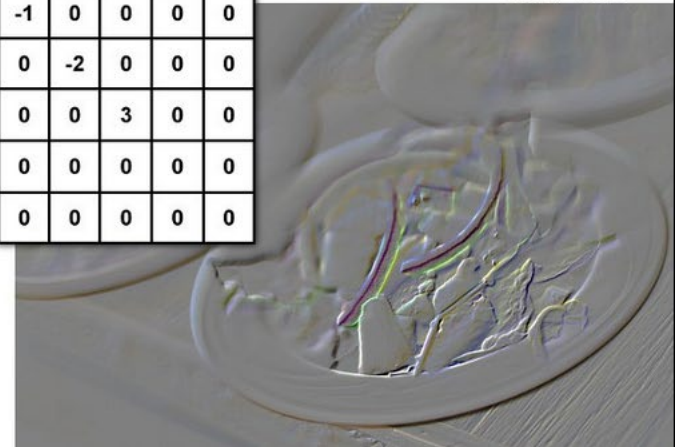
0	-1	0
-1	5	-1
0	-1	0

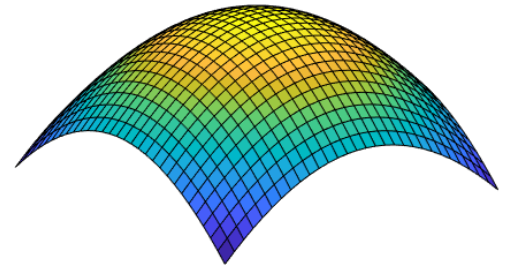
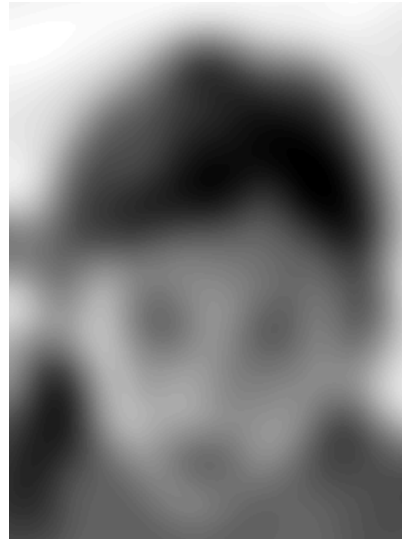
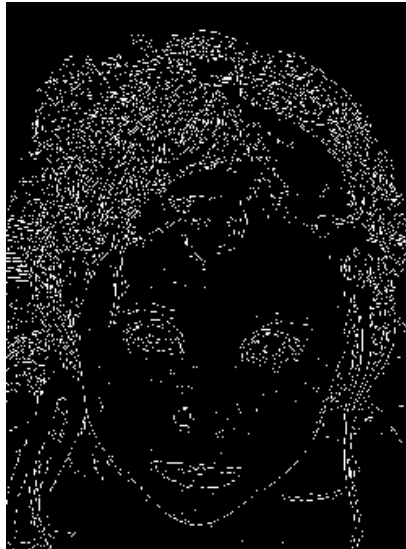
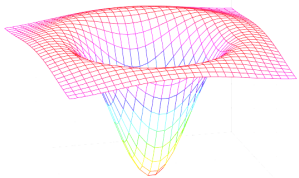
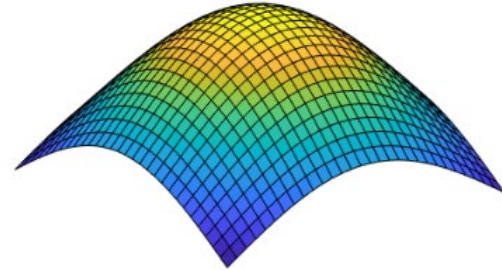
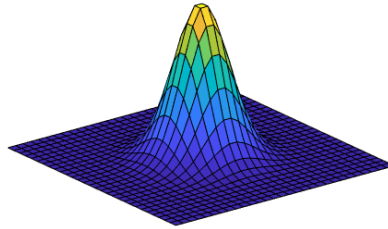
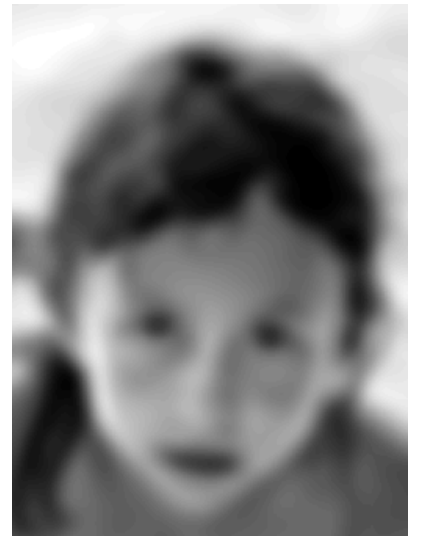
Sharpen



-1	0	0	0	0
0	-2	0	0	0
0	0	3	0	0
0	0	0	0	0
0	0	0	0	0

Emboss





Hranice

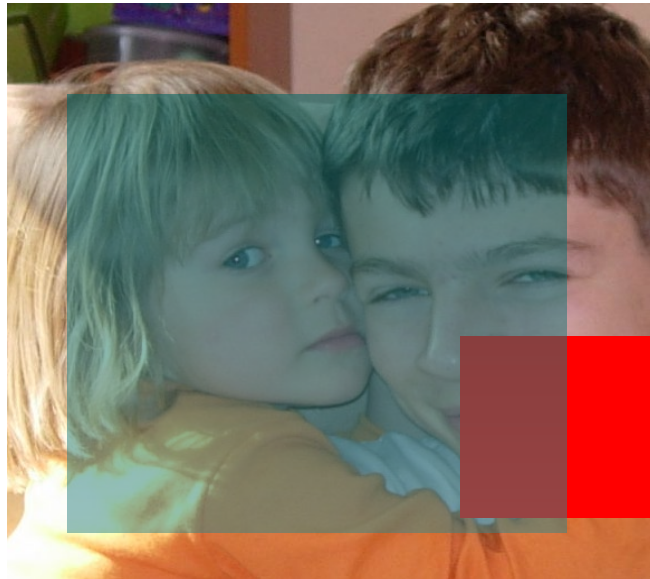


Convolution
mask



Hranice





Type
“valid”



Type
“same”

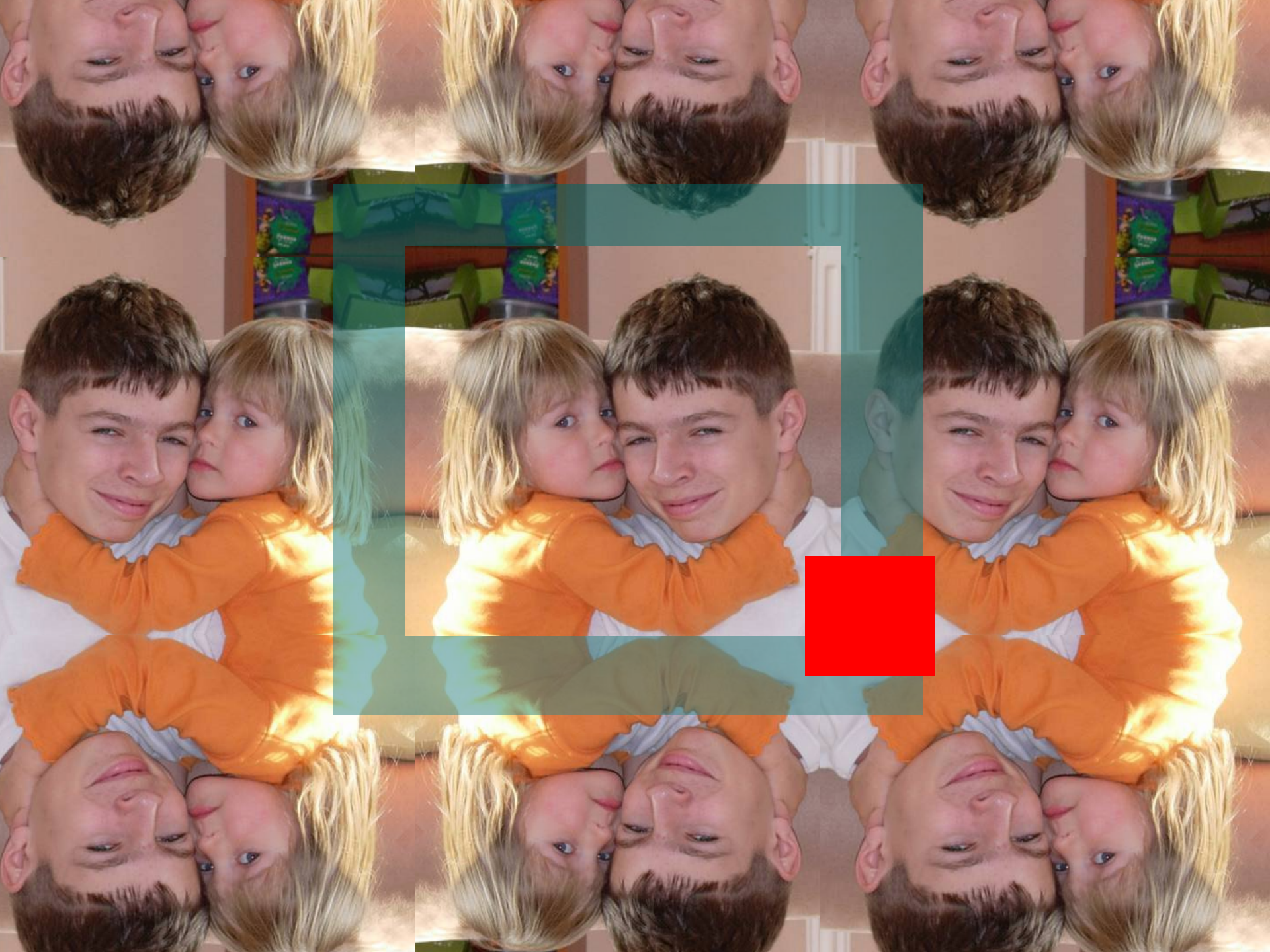


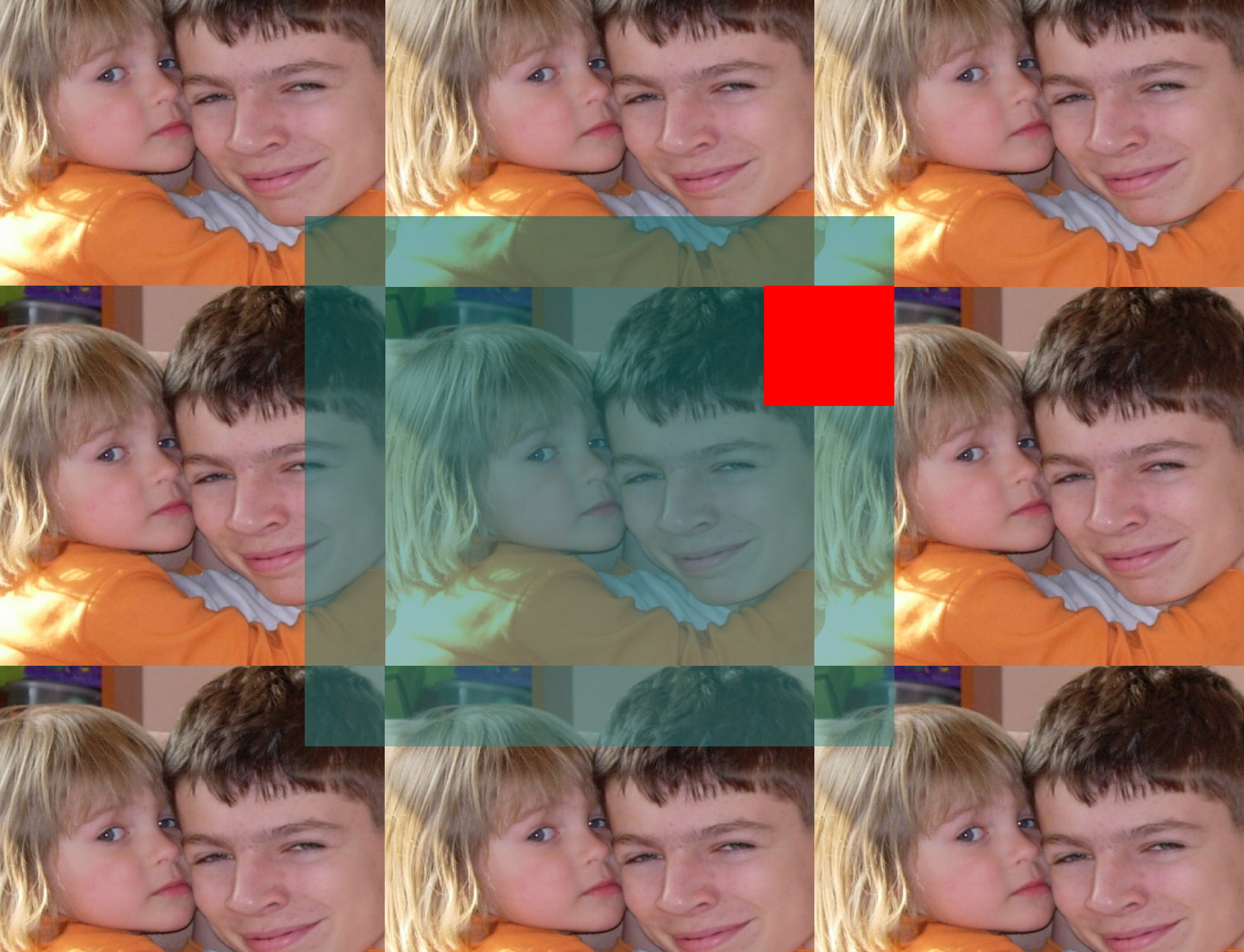
Type
“full”

Ošetření hranice

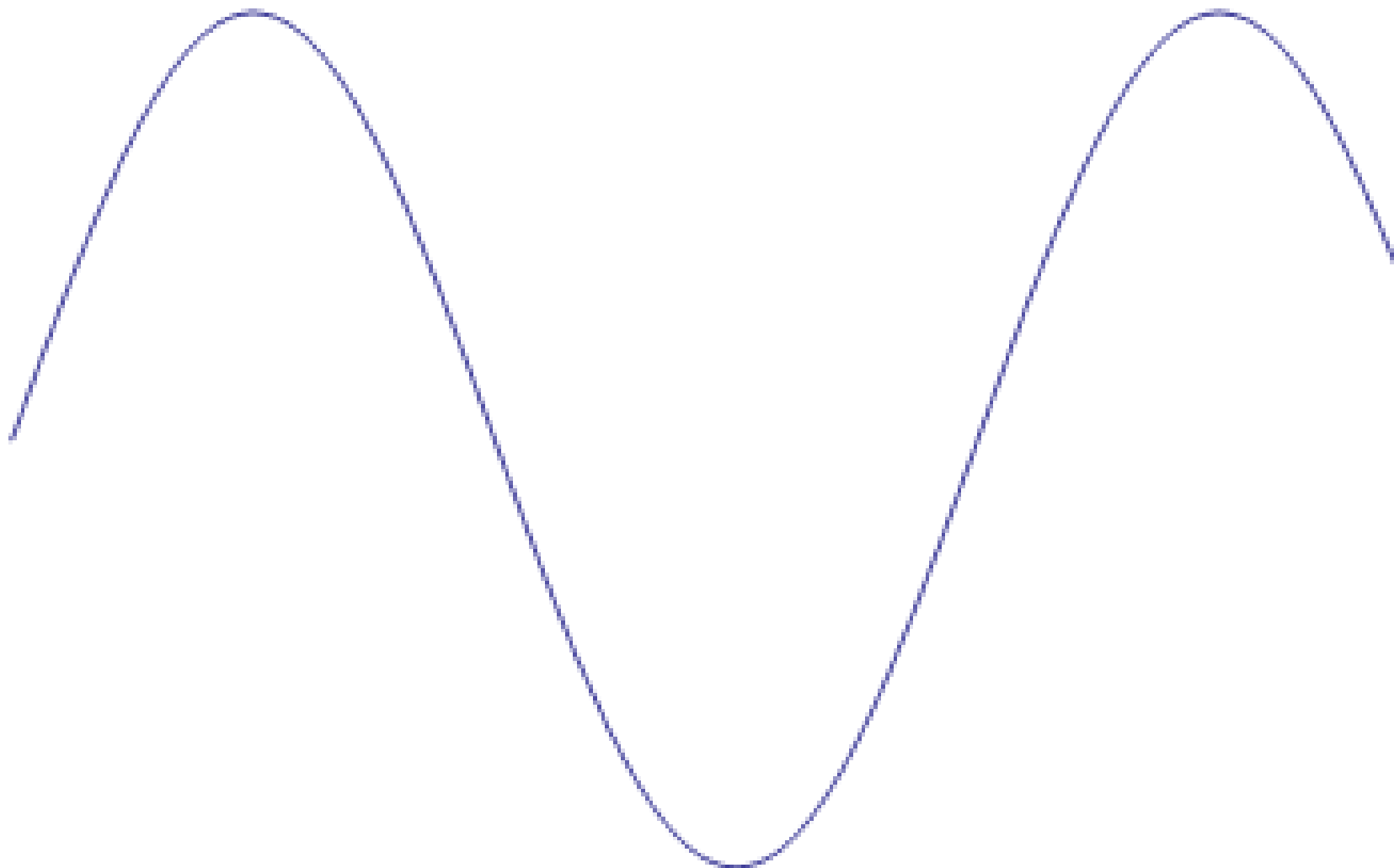
- Zero padding
- Mirror extension
- Periodic extension





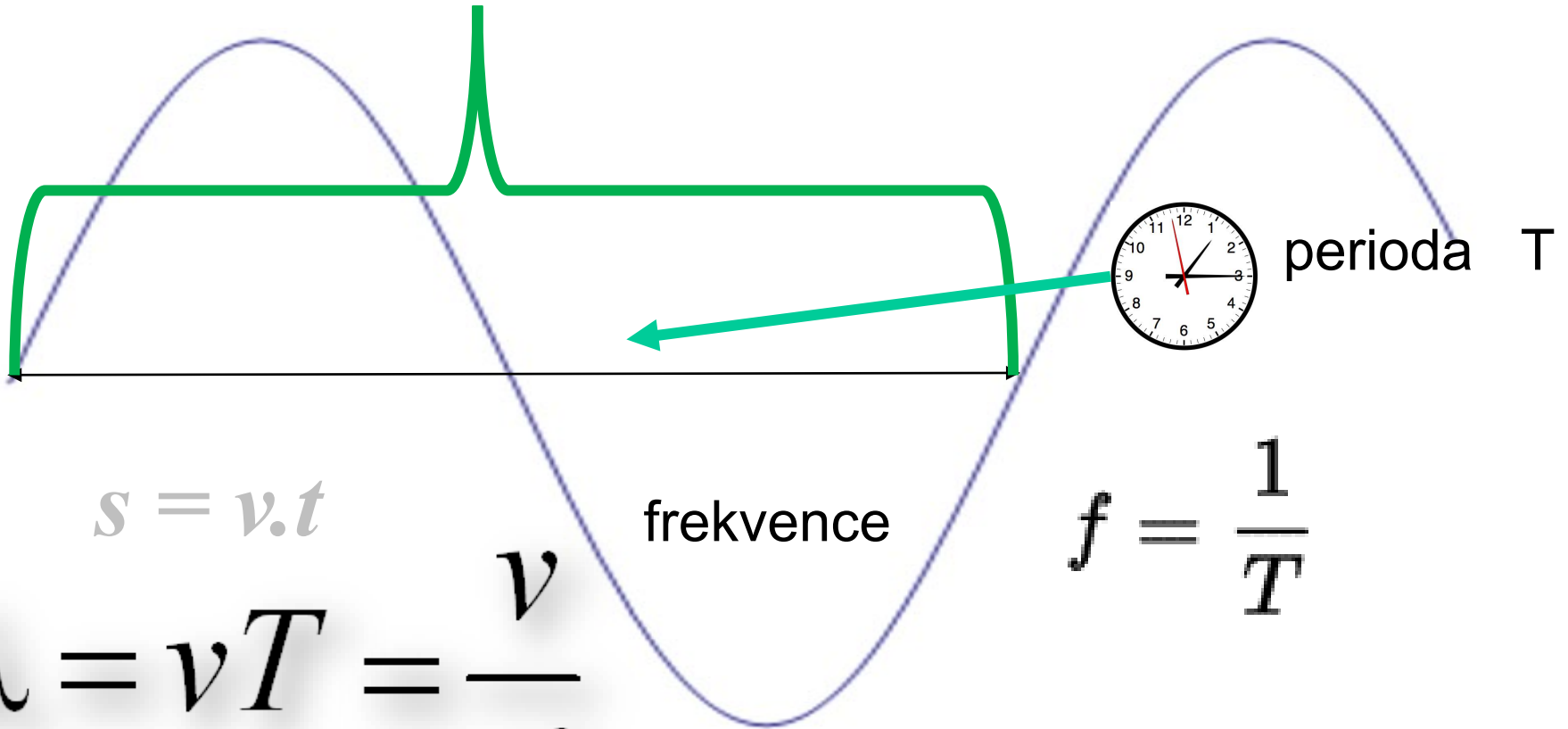


Vlnění



Vlnění

vlnová délka – λ , nejkratší vzdálenost dvou bodů kmitajících se stejnou fází



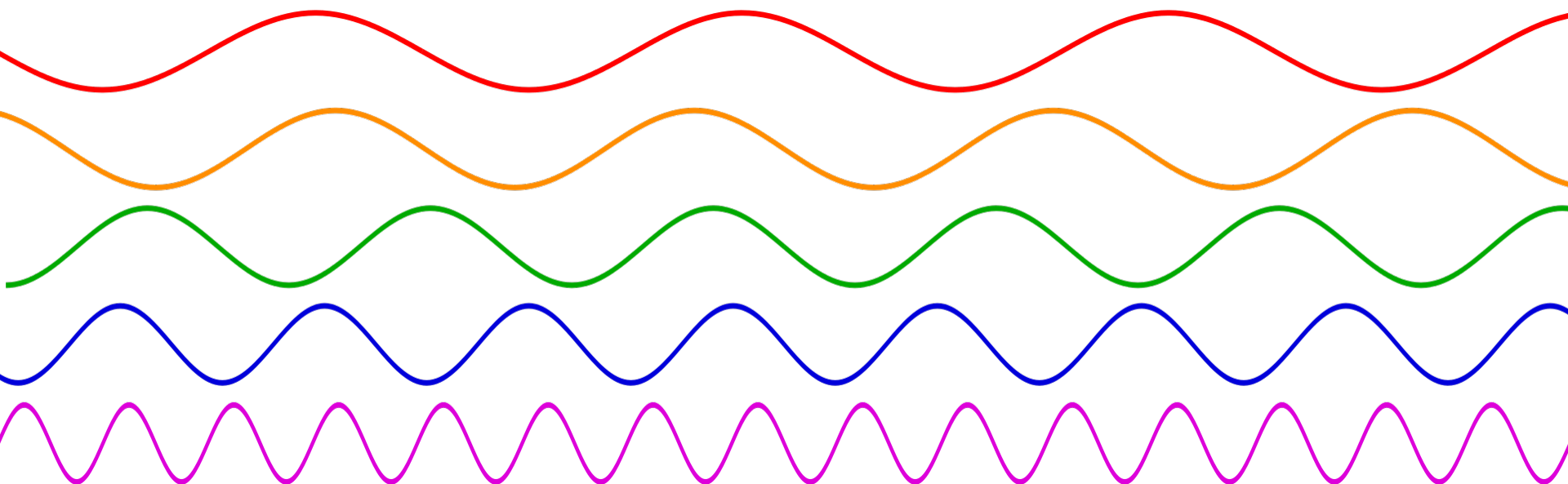
$$s = v \cdot t$$

$$\lambda = vT = \frac{v}{f}$$

frekvence

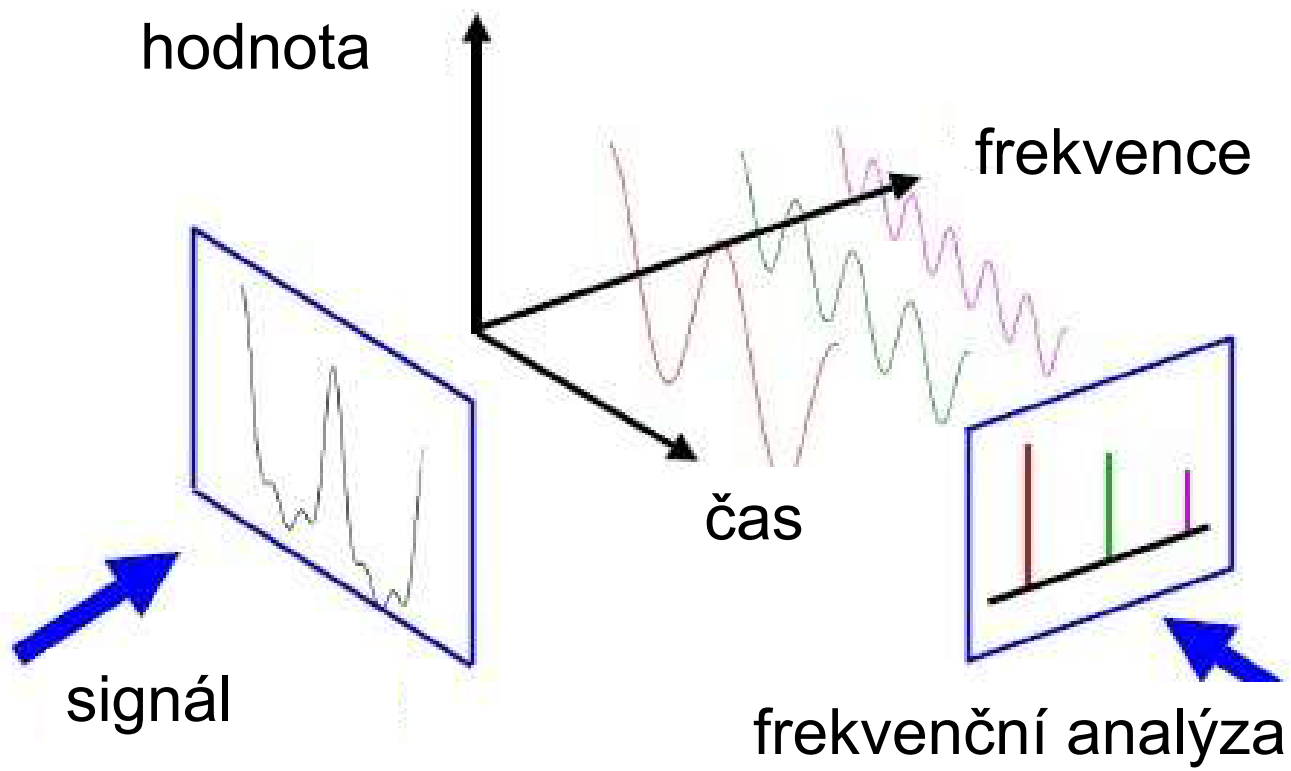
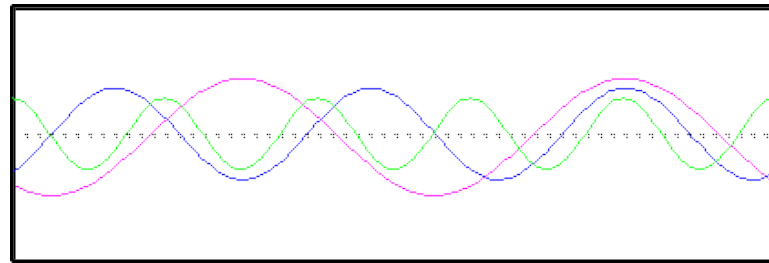
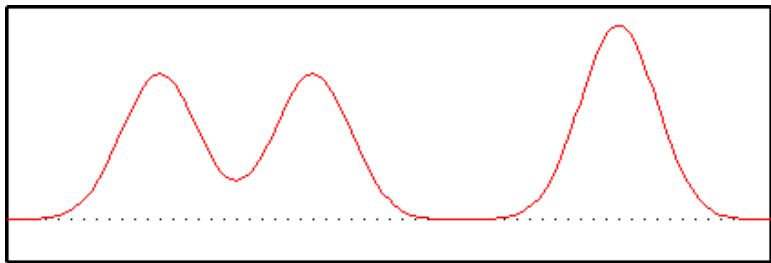
$$f = \frac{1}{T}$$

Vlnění

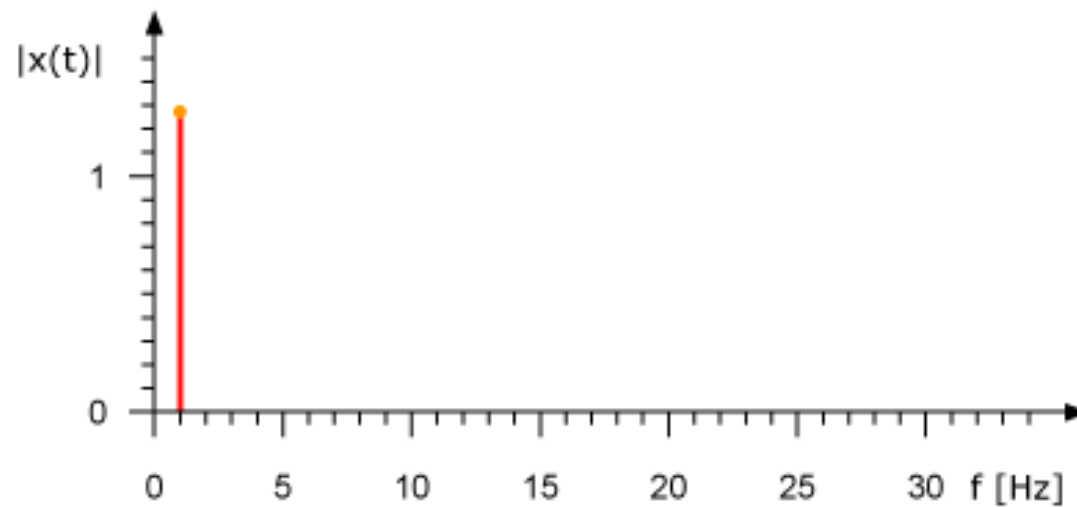
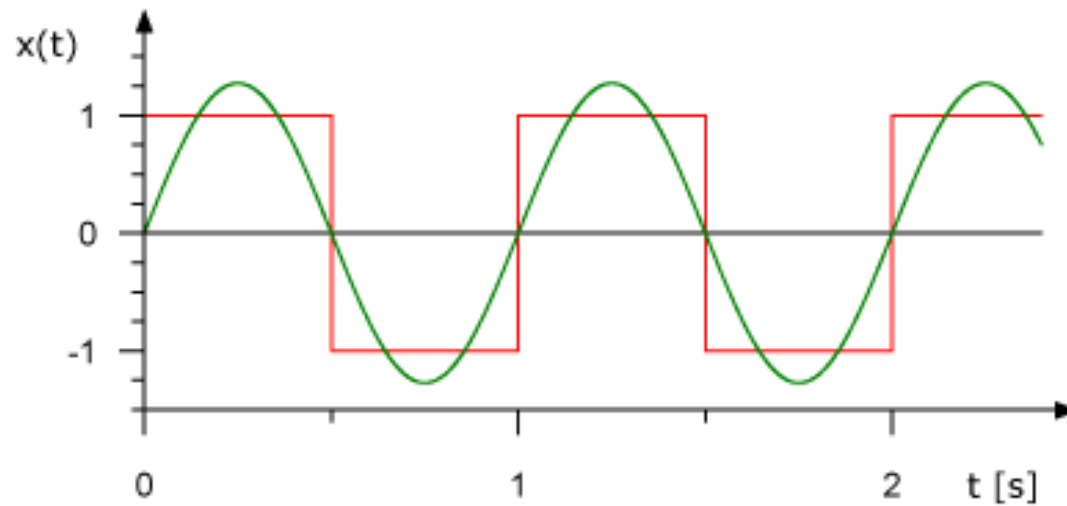


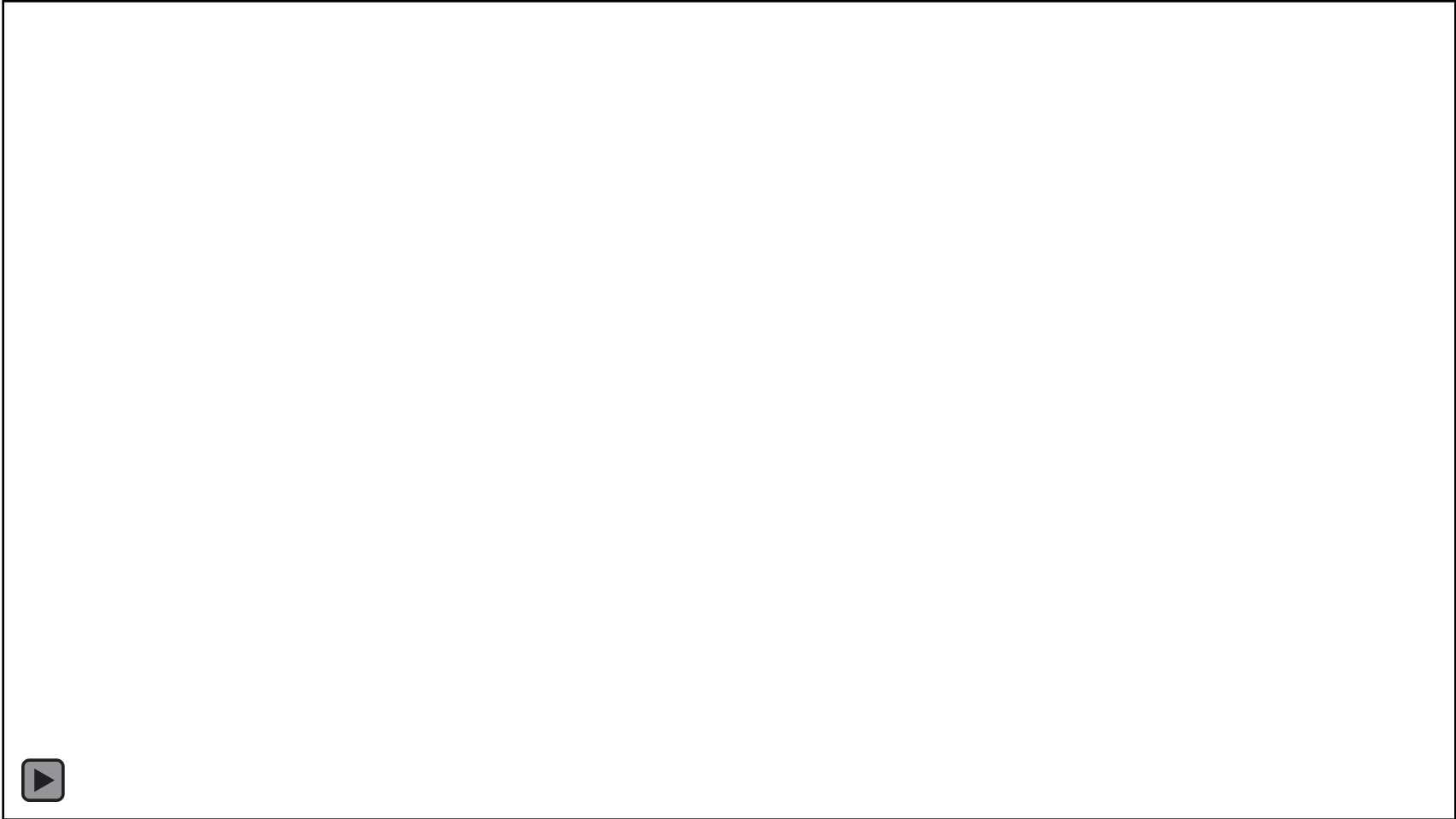
frekvence $f = \frac{1}{T}$

Vlnění

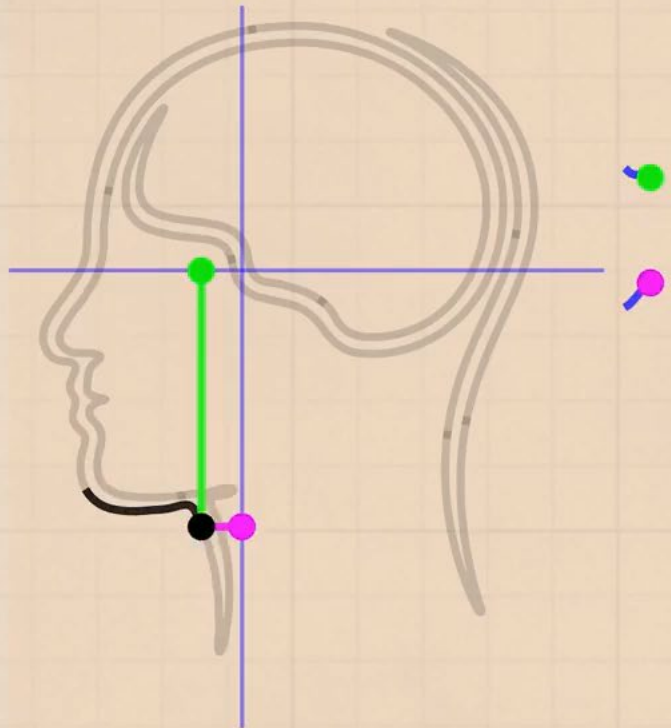


Vlnění





https://www.youtube.com/watch?v=LznjC4Lo7lE&feature=emb_title



Smarter Every Day

@BilimNeGuzelLan

Fourier transform

Fourierovy řady

$\{\phi_k\}$ – UONS v \mathcal{H} , $f \in \mathcal{H}$

úplný ortonormální systém

$c_k = (f, \phi_k)$ – Fourierovy koeficienty

$$f = \sum_{i=1}^{\infty} c_k \phi_k$$



$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

Fourier transform

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx.$$

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$$

Existence FT – stačí $f \in L_1$

$$F = \mathcal{R}(F) + i\mathcal{I}(F) = |F| \cdot e^{i \cdot \text{ph}(F)}$$

Vlastnosti FT

- linearity

$$\mathcal{F}[af(x) + bg(x)] = a\mathcal{F}[f(x)] + b\mathcal{F}[g(x)] = aF(k) + bG(k).$$

- convolution

convolution theorem

$$\mathcal{F}[f]\mathcal{F}[g] = \mathcal{F}[f * g]$$

- shift

shift theorem

$$\mathcal{F}_x[f(x - x_0)](k) = e^{-2\pi i k x_0} F(k).$$

- rotation

rotational theorem

$$\mathcal{F}\{f(R_\theta \mathbf{r})\} = F(R_\theta \mathbf{k})$$

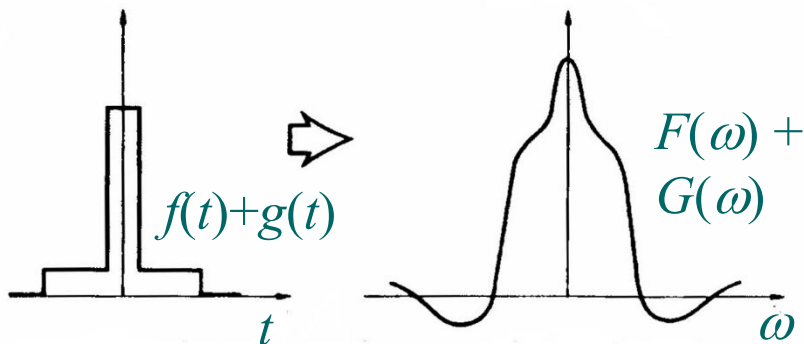
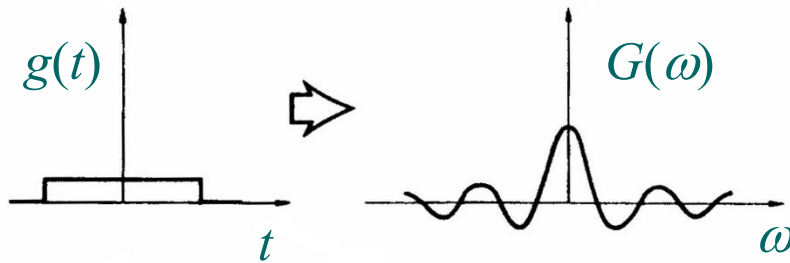
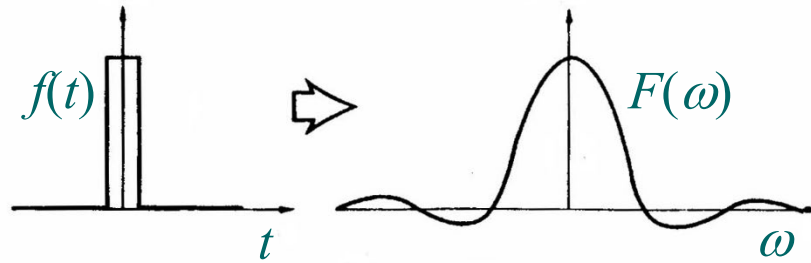
- scaling

similarity theorem

$$\mathcal{F}_x[f(ax)](k) = |a|^{-1} F\left(\frac{k}{a}\right).$$

linearita

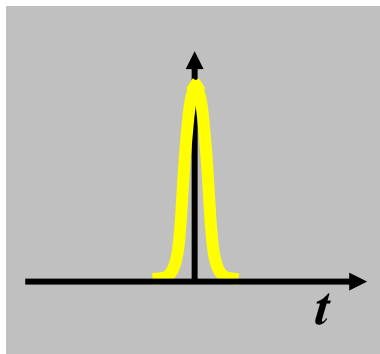
$$\mathcal{F}[af(x) + bg(x)] = a\mathcal{F}[f(x)] + b\mathcal{F}[g(x)] = aF(k) + bG(k).$$



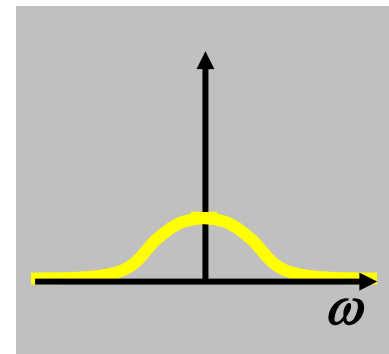
scaling
similarity theorem

$$\mathcal{F}_x[f(ax)](k) = |a|^{-1} F\left(\frac{k}{a}\right) \cdot \text{Short pulse}$$

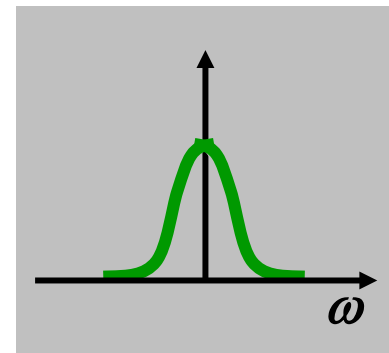
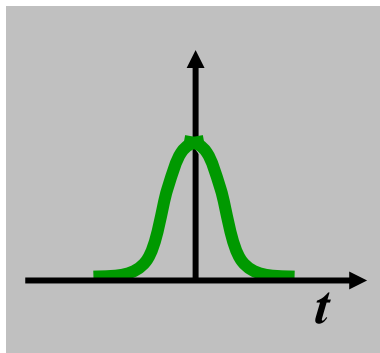
$f(t)$



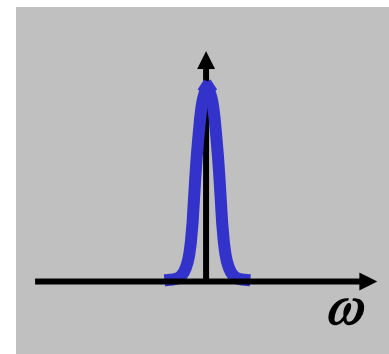
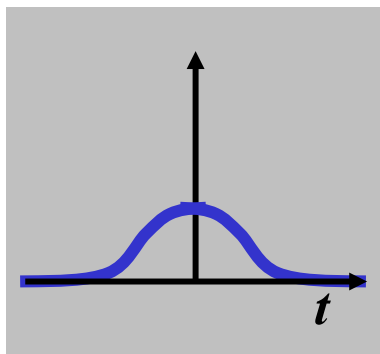
$F(\omega)$



Medium-length pulse



Long pulse



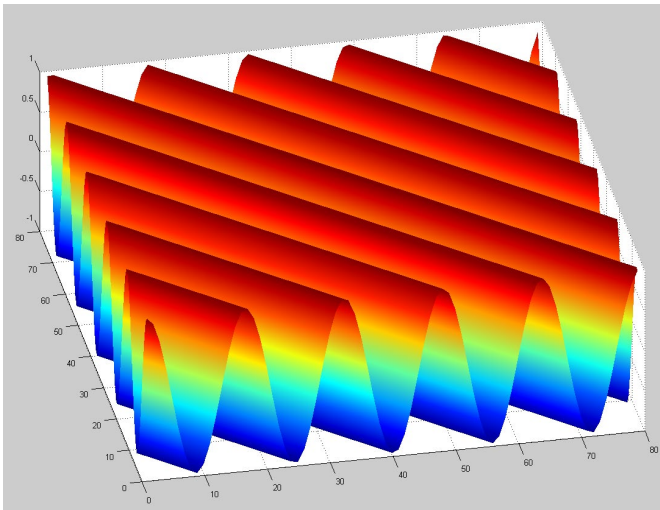
2D Fourier transform

$$\{\phi_{uv}(x, y)\} = \{e^{-2\pi i(ux+vy)}\}$$

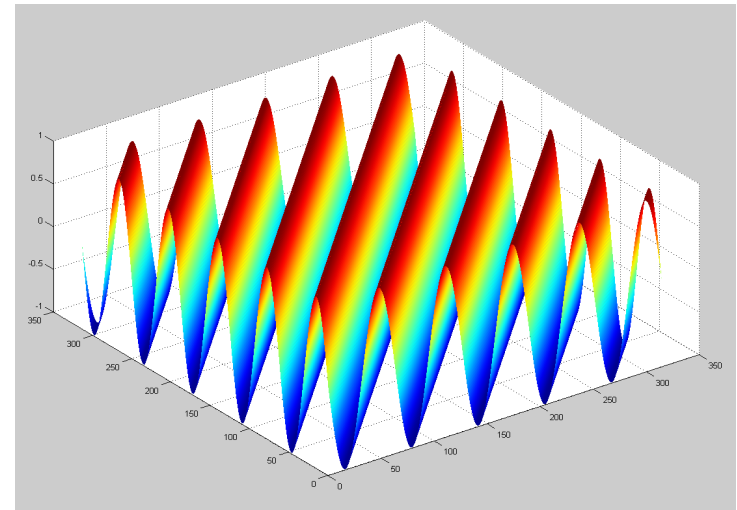
$$F(u, v) = \int \int_{\mathbb{R}^2} f(x, y) e^{-2\pi i(ux+vy)} dx dy$$

$$f(x, y) = \int \int_{\mathbb{R}^2} F(u, v) e^{-2\pi i(ux+vy)} du dv$$

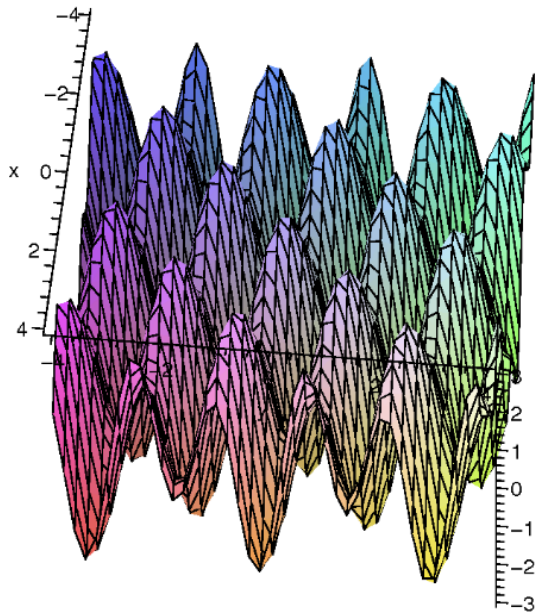
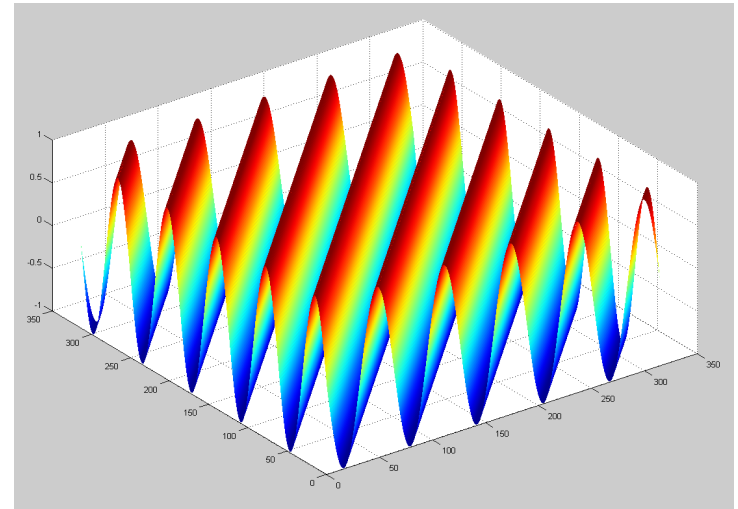
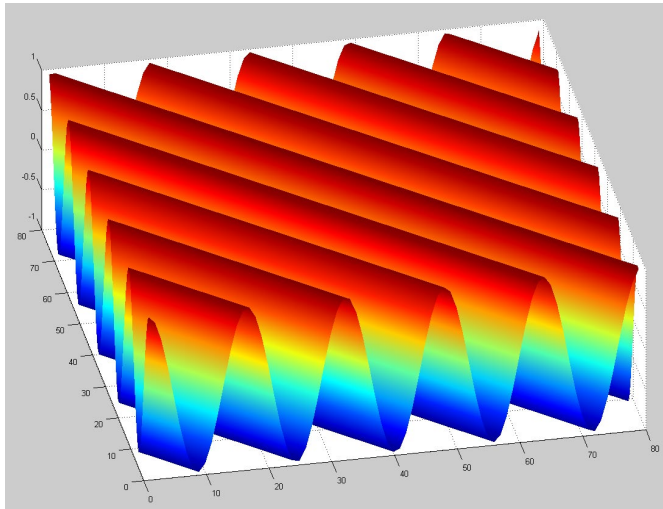
Basis functions of 2D FT



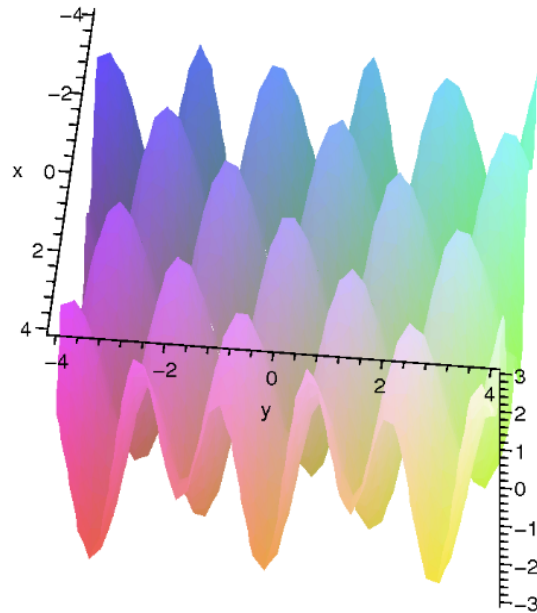
real, $u=v$



imag, $u=v$



$\sin(3x + 2y) + \cos(x + 4y)$

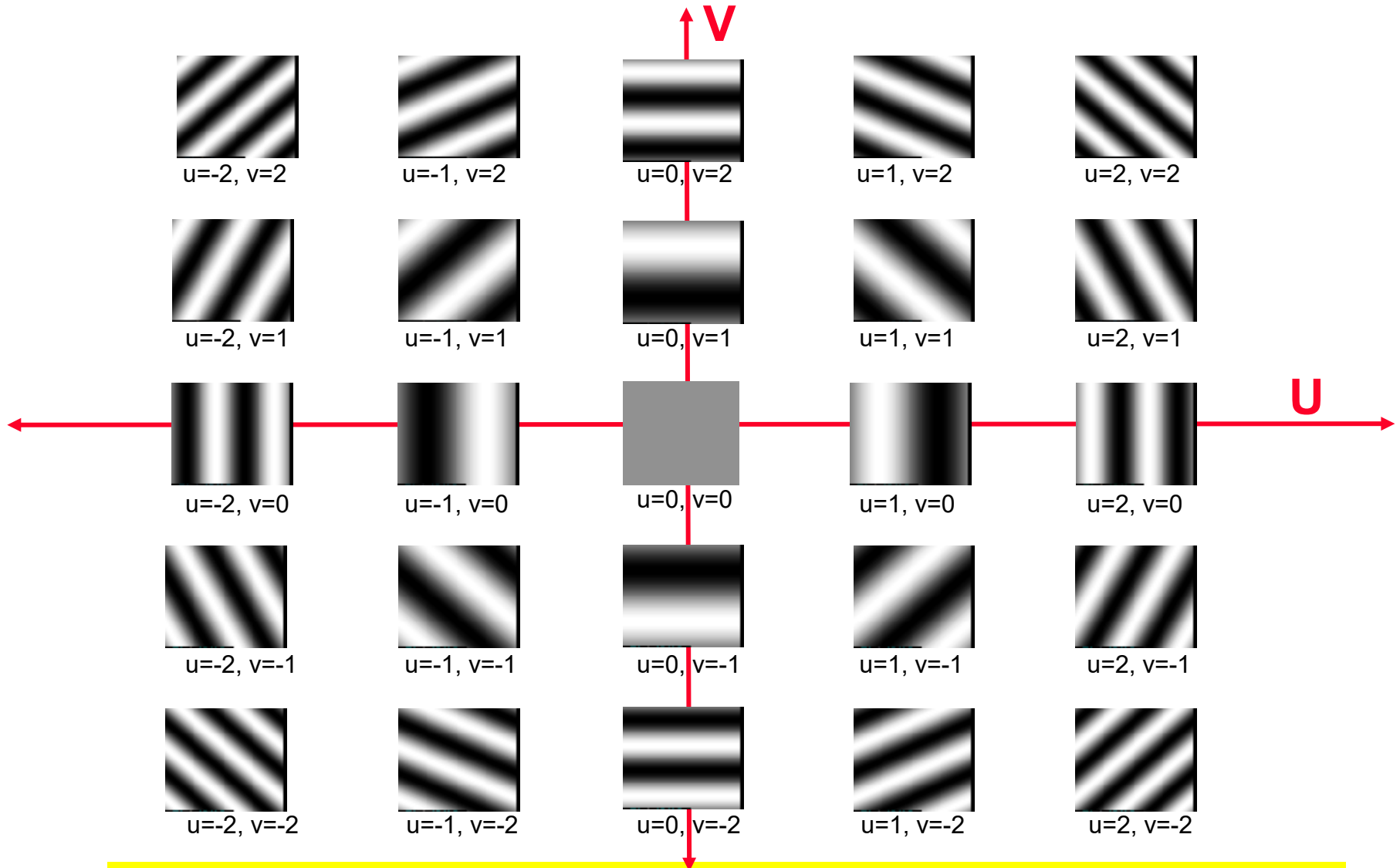


different display only



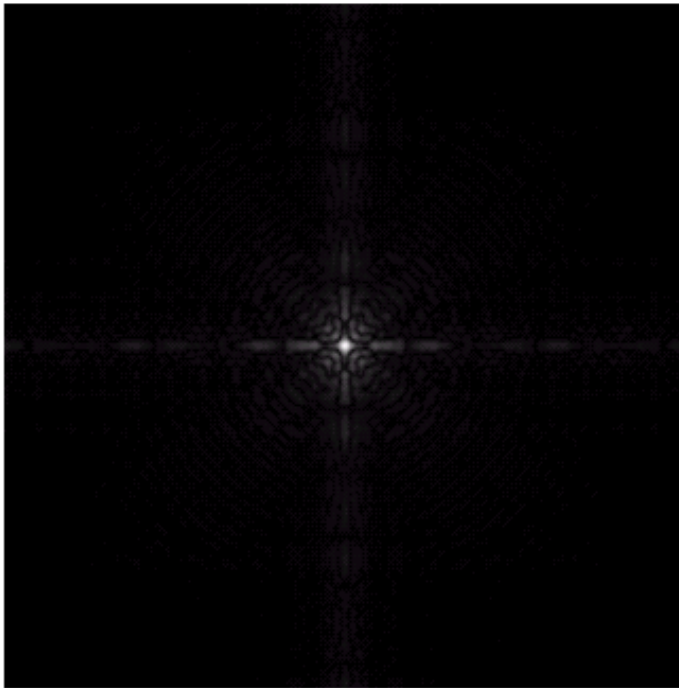
analogy: carton egg tray

The 2D Basis Functions $e^{2\pi i(ux+vy)}$

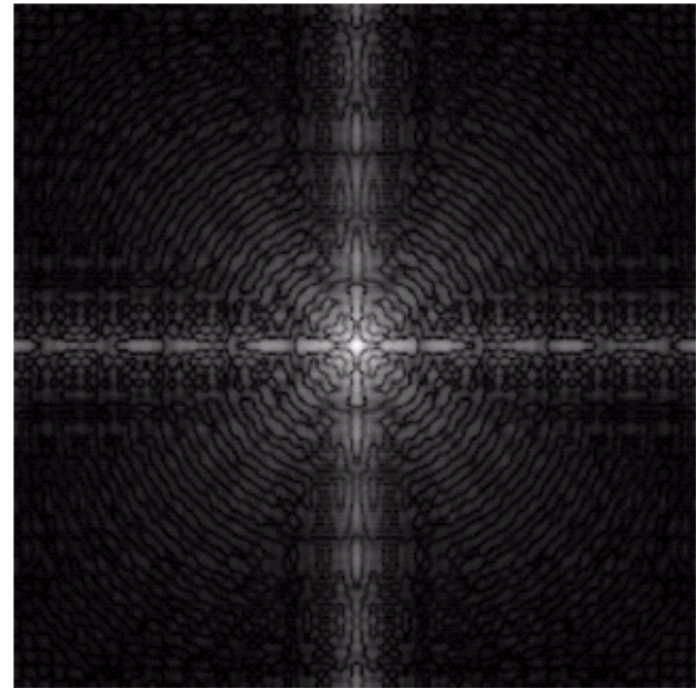


The wavelength is $1/\sqrt{u^2 + v^2}$. The direction is u/v .

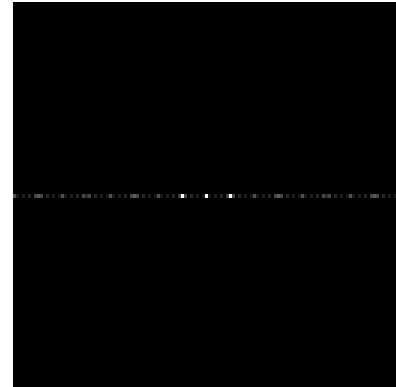
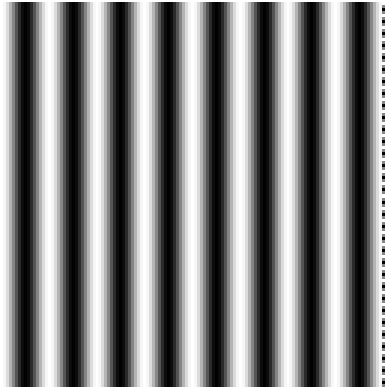
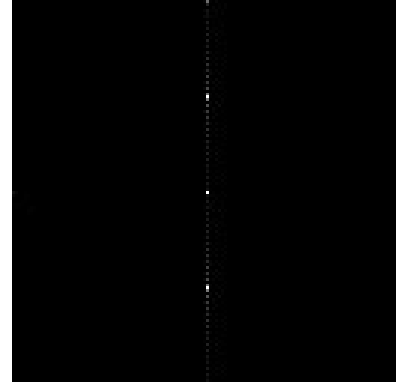
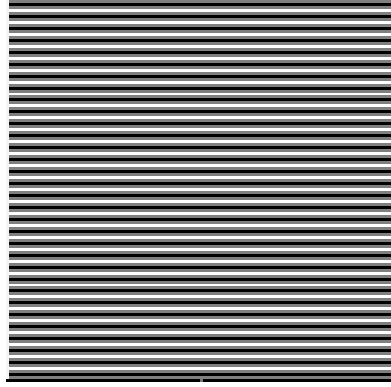
Zobrazení spektra



ampl



$\log(\text{ampl} + 1)$



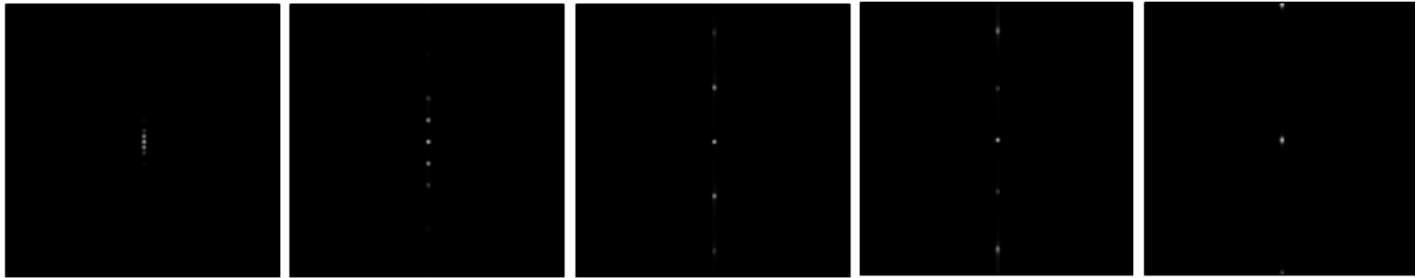
$f = 1$

$f = 4$

$f = 10$

$f = 20$

$f = 25$

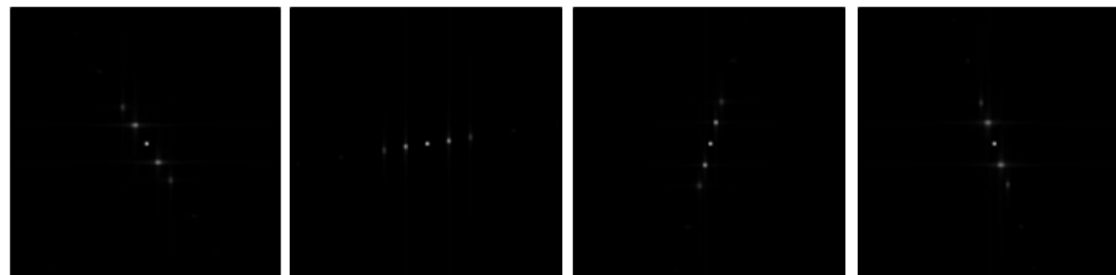
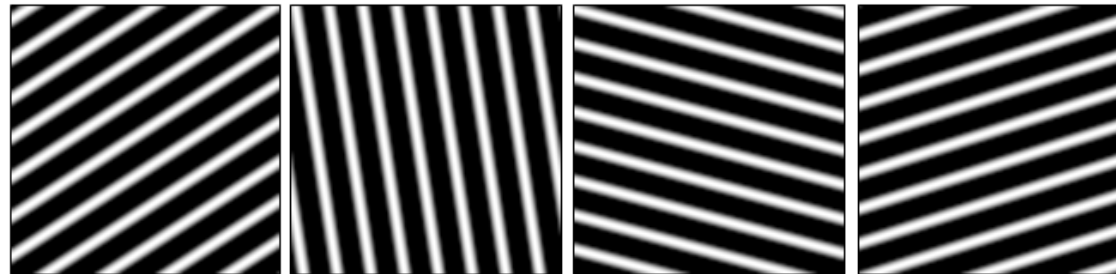


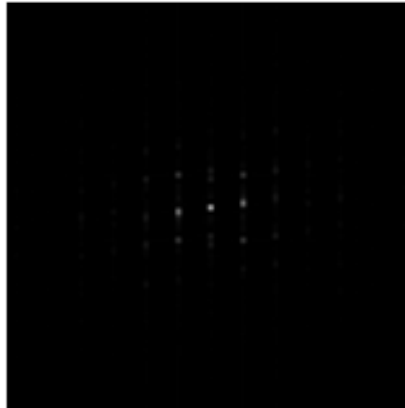
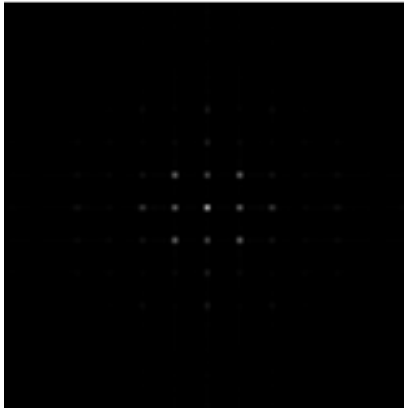
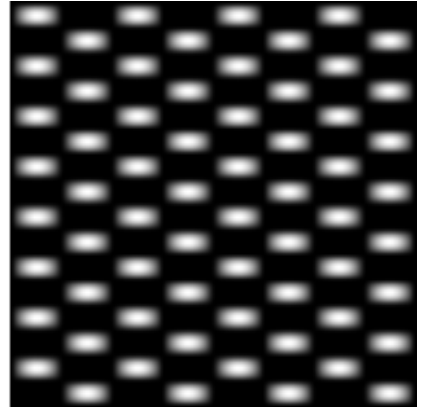
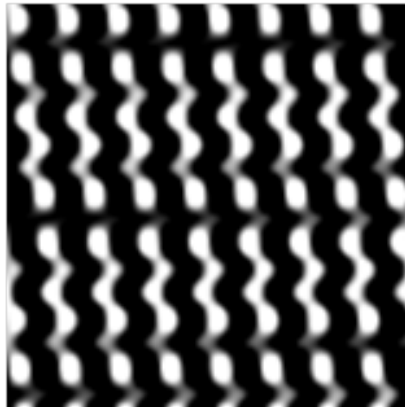
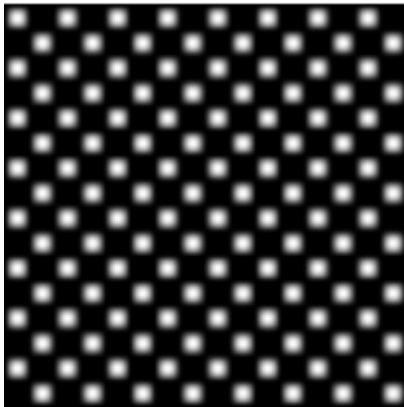
$\theta = 10$

$\theta = 30$

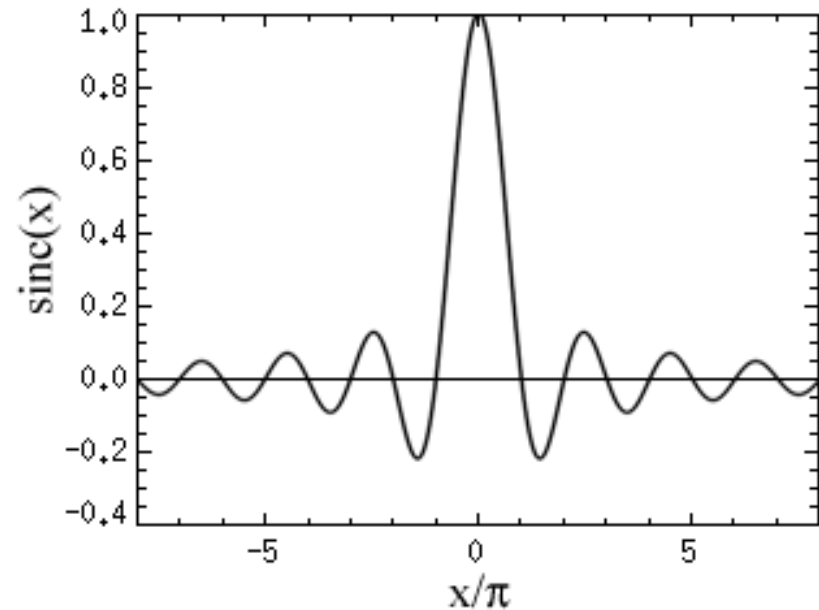
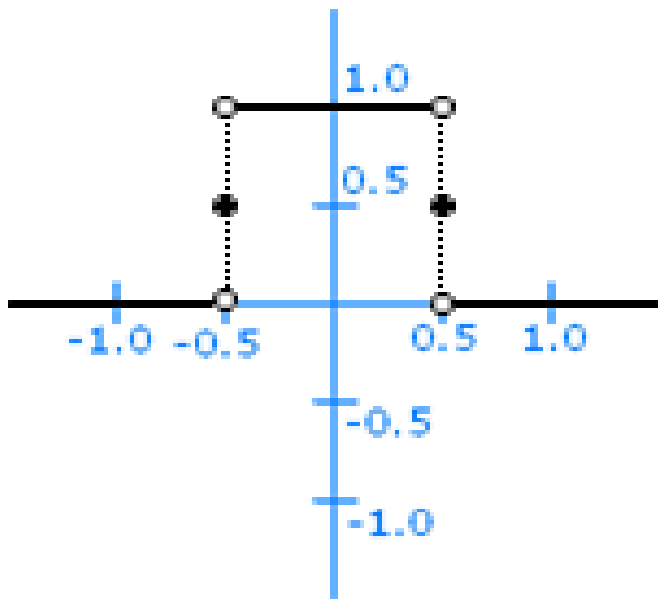
$\theta = 50$

$\theta = 60$





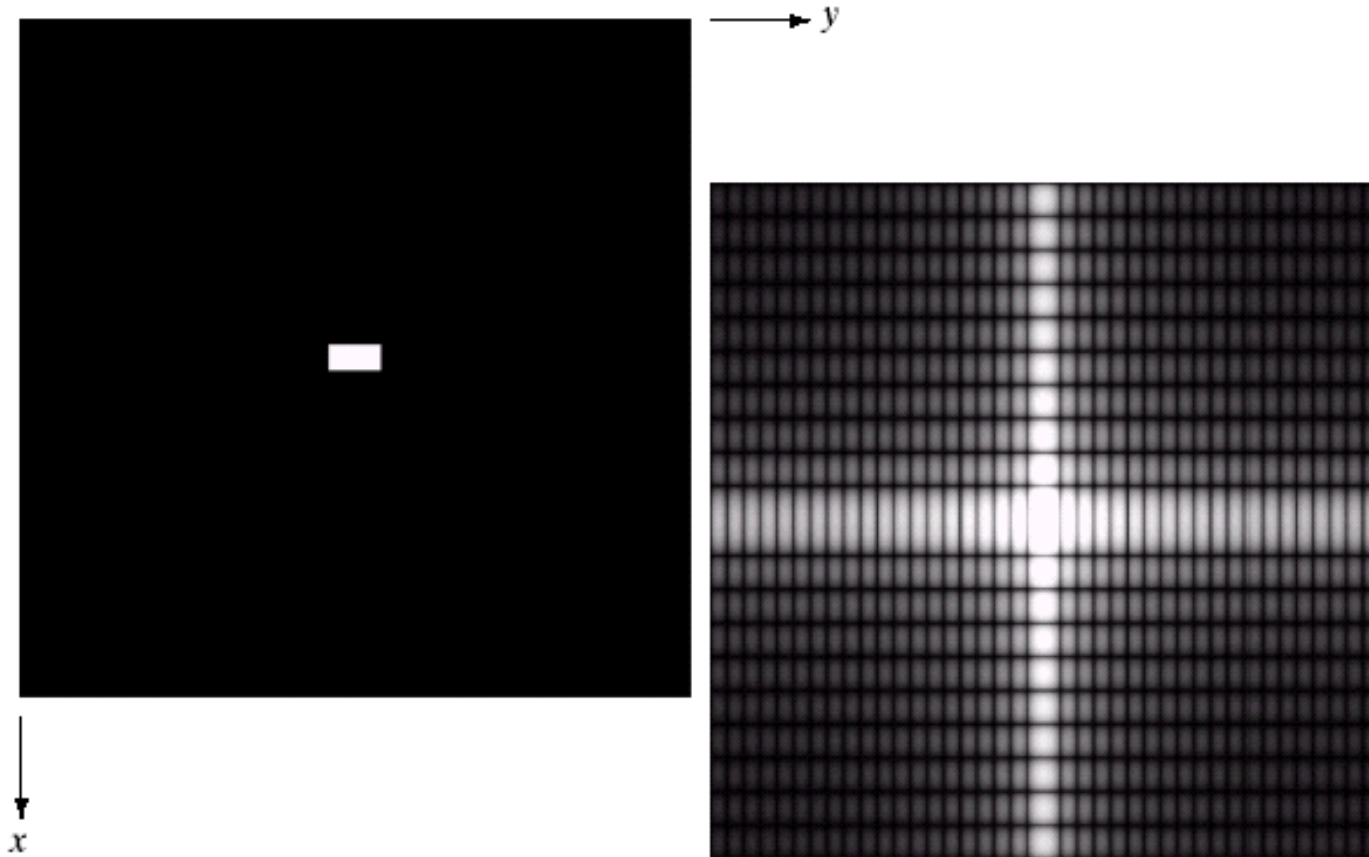
Obdélníkový puls

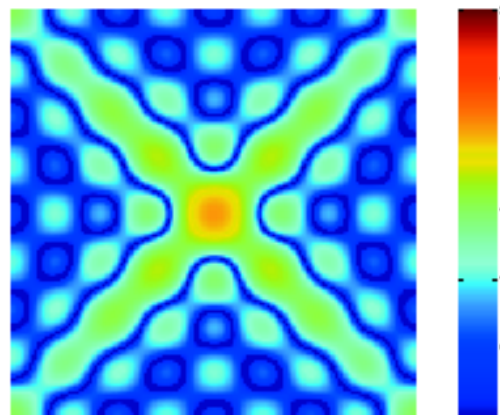
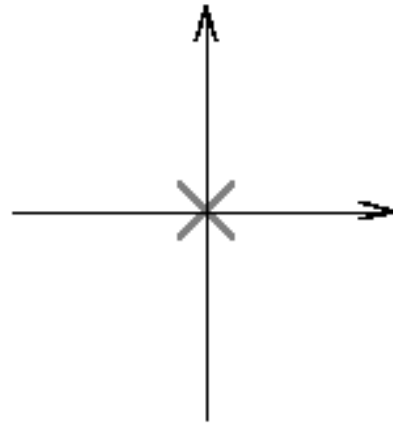
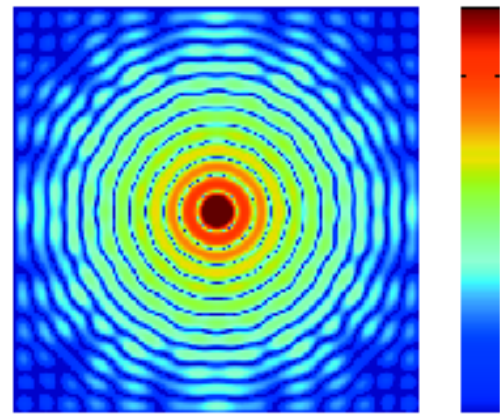
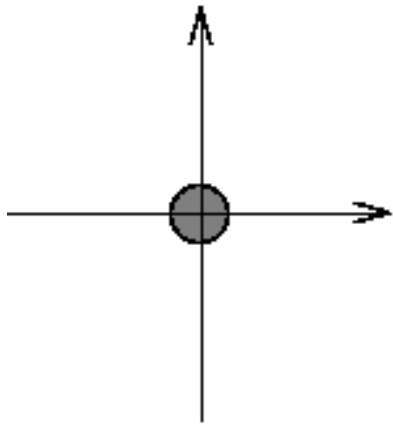
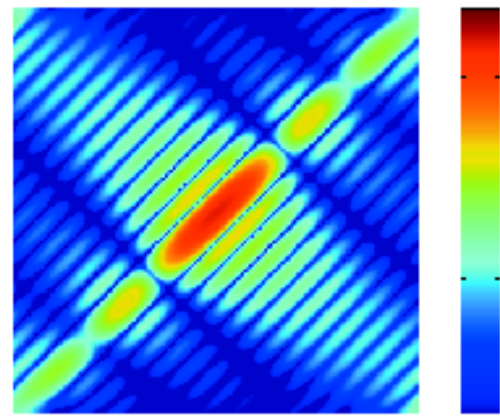
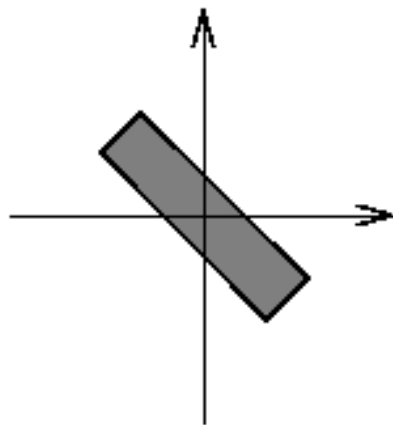


$$\Pi(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

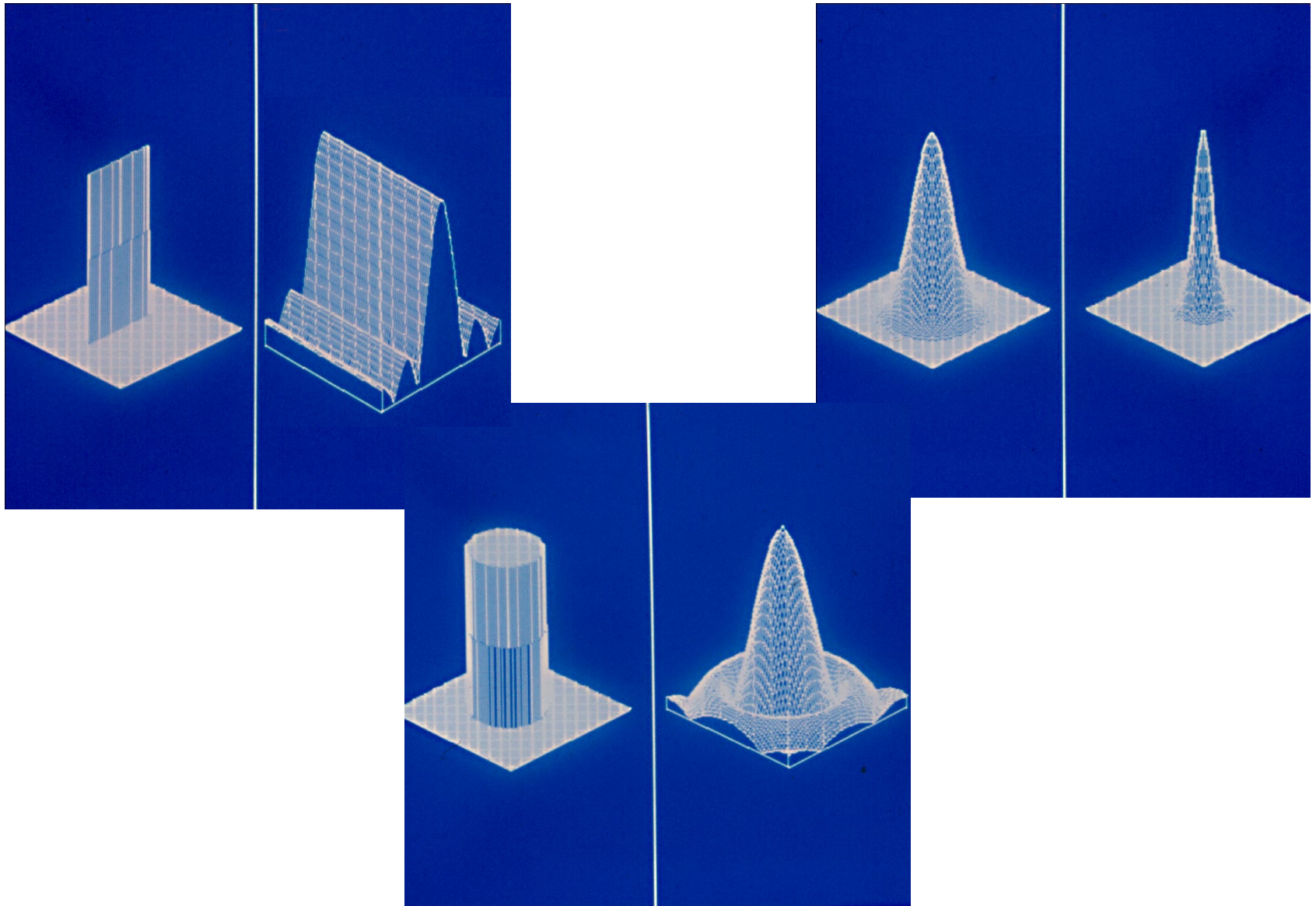
$$\mathcal{F}\{\Pi(t)\}(x) = \frac{\sin(\pi x)}{\pi x} = \text{sinc}(x)$$

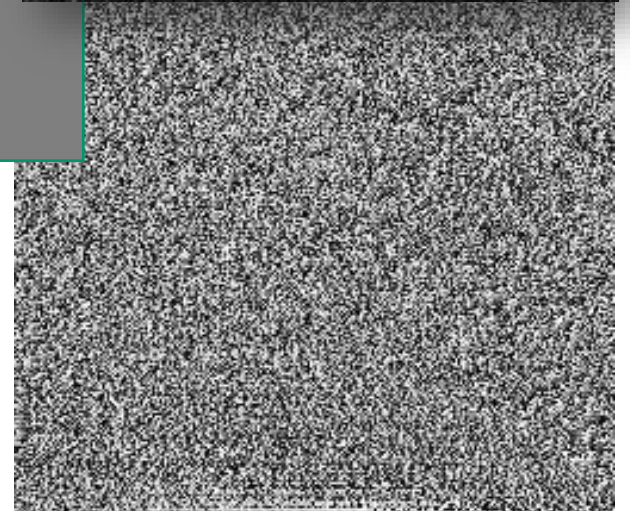
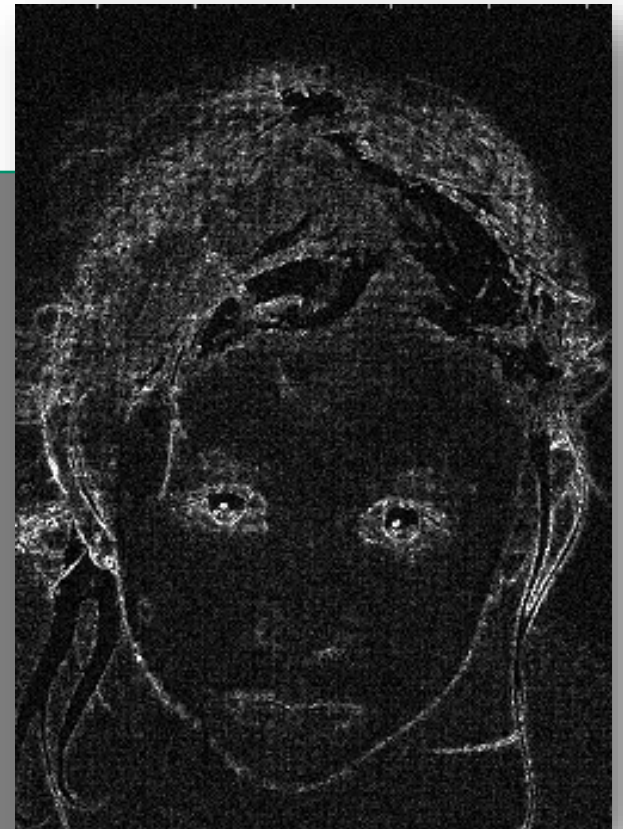
Obdélníkový puls 2D





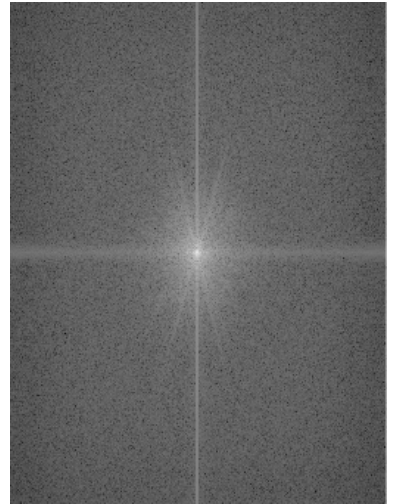
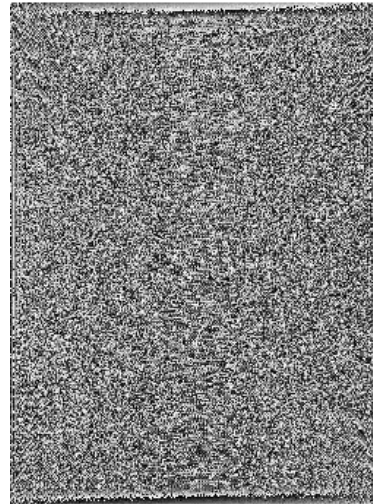
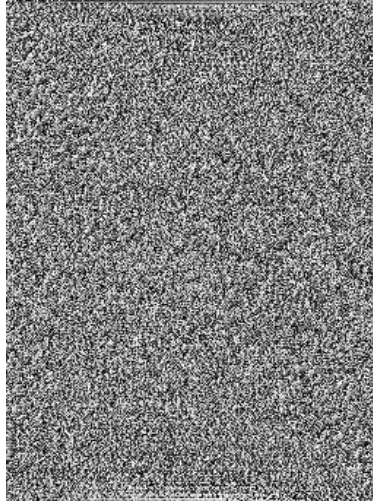
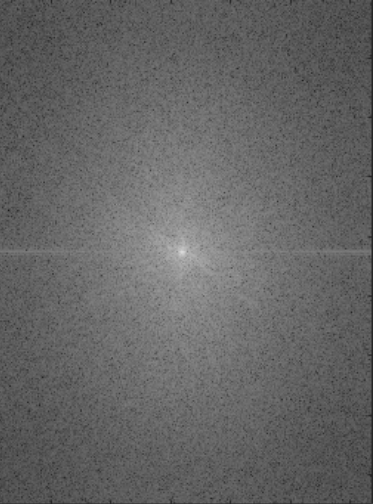
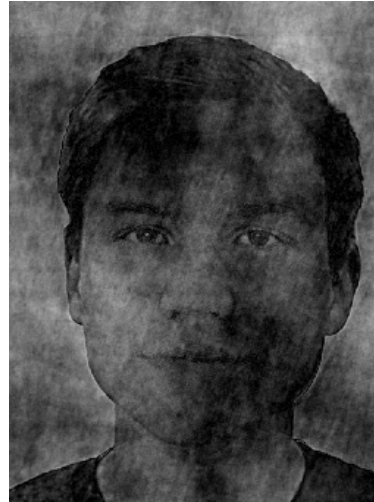
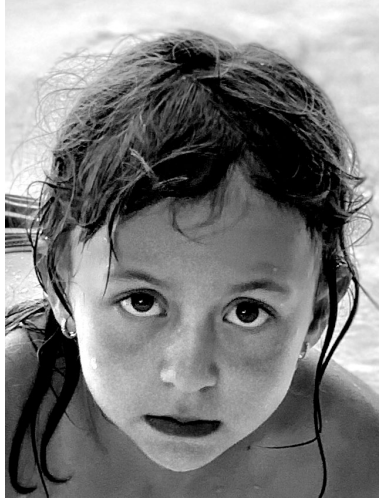
Další ukázky

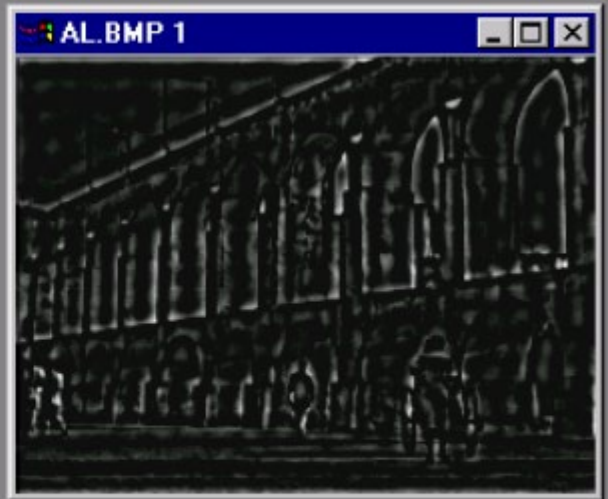
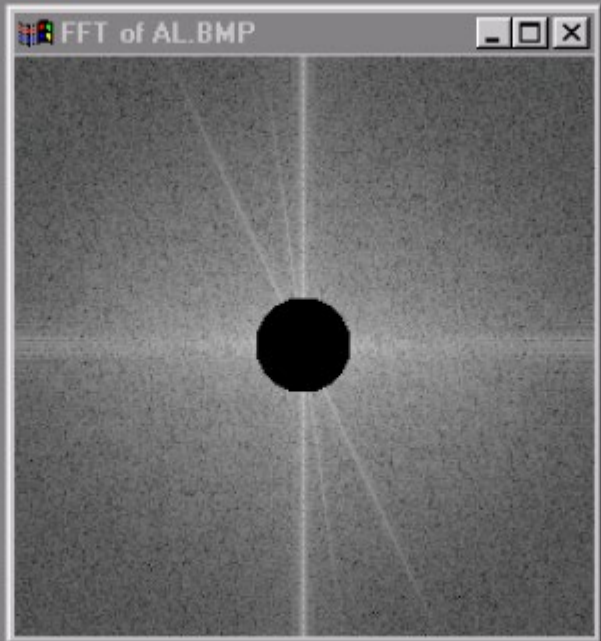
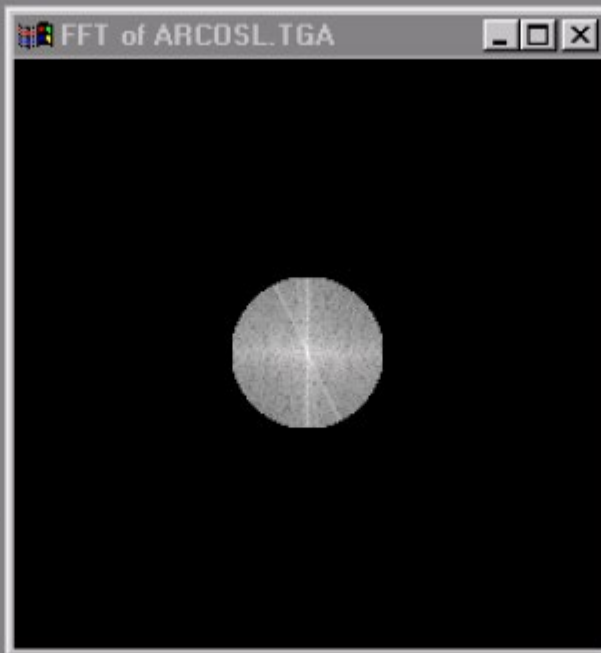




$$F(u, v) = \int \int_{\mathbb{R}^2} f(x, y) e^{-2\pi i(ux+vy)} dx dy$$

$$F(u, v) = |F(u, v)| e^{i\phi(u, v)}$$





Otázky

- 1) Proofs & calculations
- 2) $FT(\delta) = ??$
- 3) Symmetry of the FT

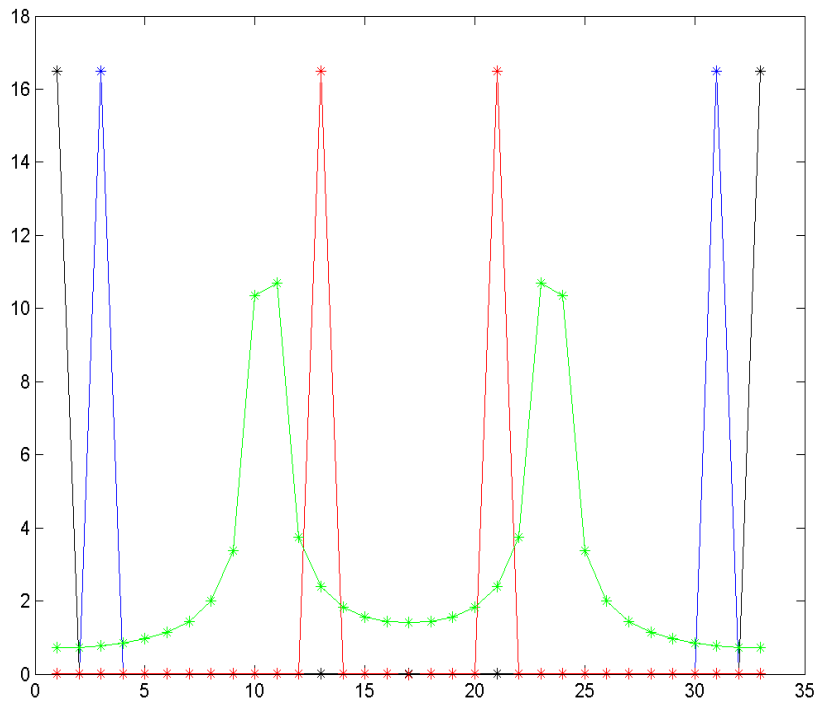
Diskrétní Fourier Transform

$$F_n = \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N}.$$

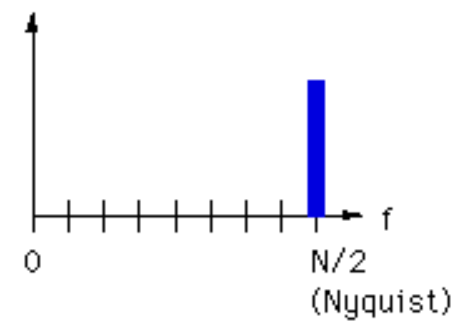
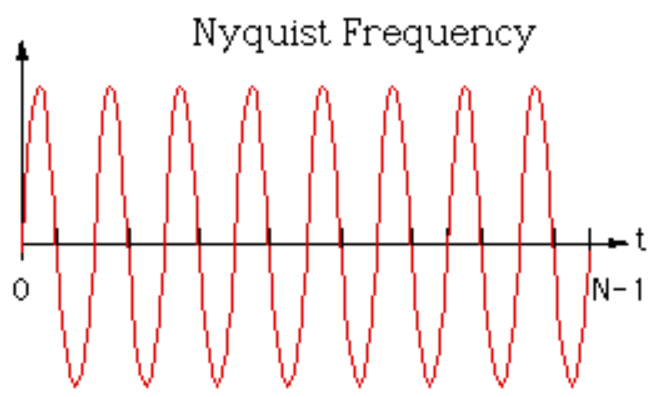
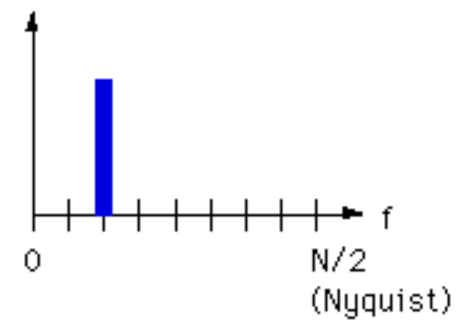
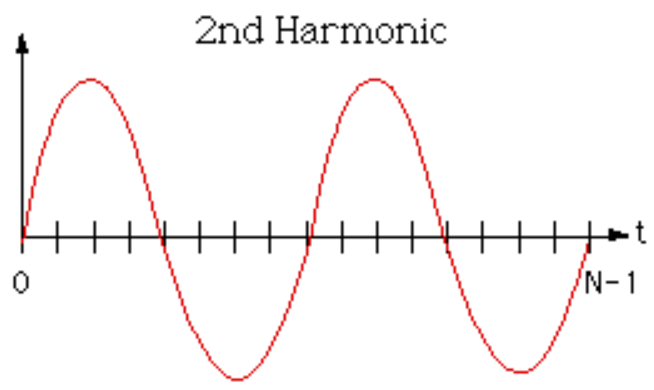
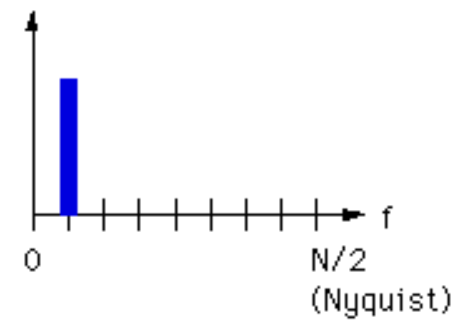
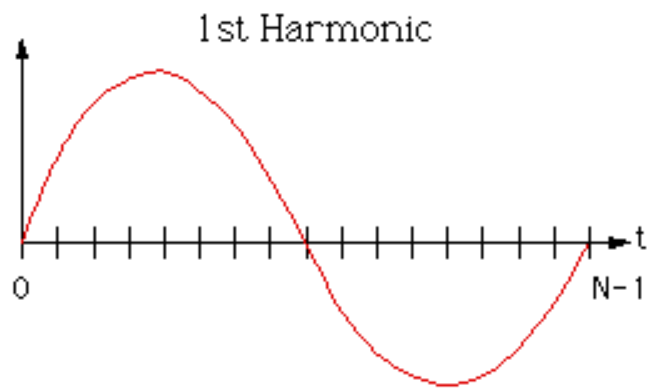
$$f_k = \frac{1}{N} \sum_{n=0}^{N-1} F_n e^{2\pi i n k / N}.$$

$$F_n \equiv \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N}.$$

$$f_k \equiv \frac{1}{N} \sum_{n=0}^{N-1} F_n e^{2\pi i n k / N}.$$



4 / N
 6.5 / N
 17 / N
 19 / N



Convolution theorem

$$\mathcal{F}\{(f * g)(t)\} = F(\omega) \cdot G(\omega)$$

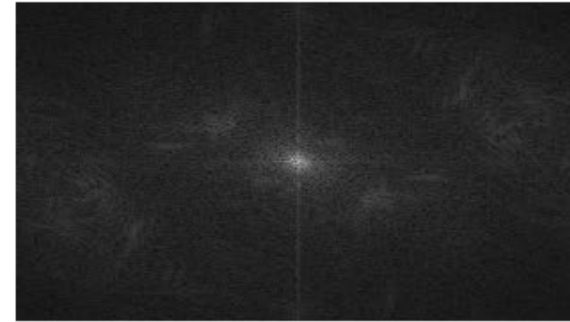
$$\mathcal{F}^{-1}\{F(\omega) \cdot G(\omega)\} = (f * g)(t)$$

Convolution theorem

$f(x,y)$



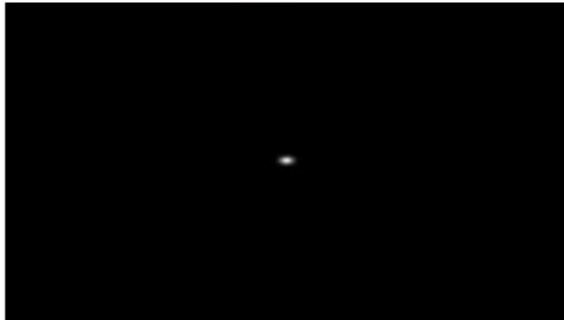
*



$|F(s_x, s_y)|$

x

$h(x,y)$



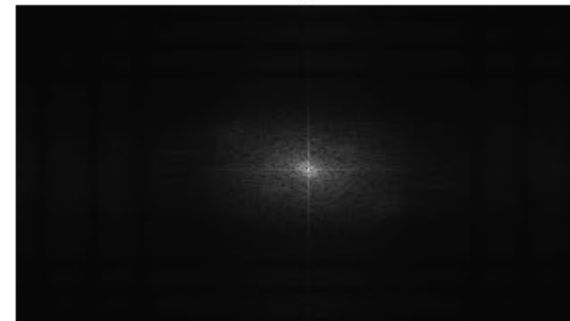
⇓



$|H(s_x, s_y)|$

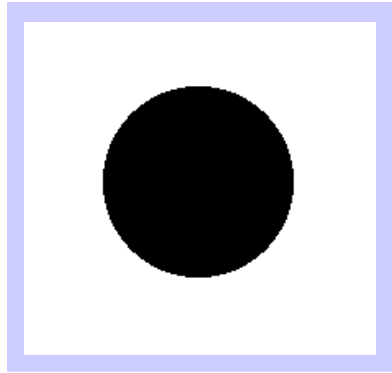
⇓

$g(x,y)$



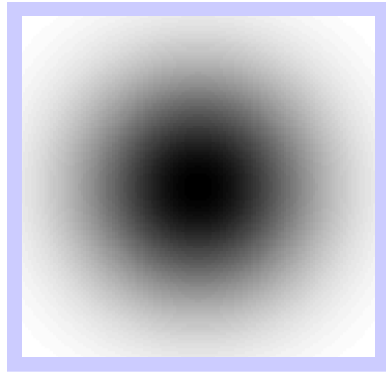
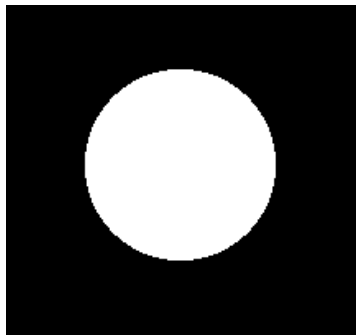
$|G(s_x, s_y)|$

Filtering in the Fourier domain



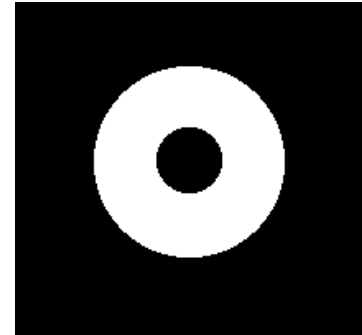
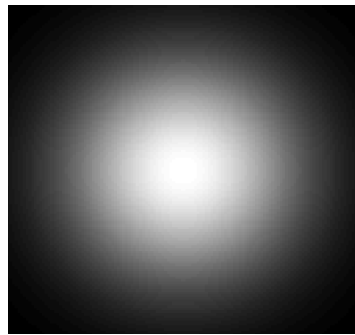
high pass

low pass

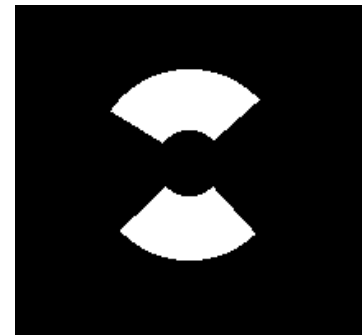


Gaussian high pass

Gaussian low pass



band pass



directional

Discrete convolution theorem



Convolution
mask



Discrete convolution theorem

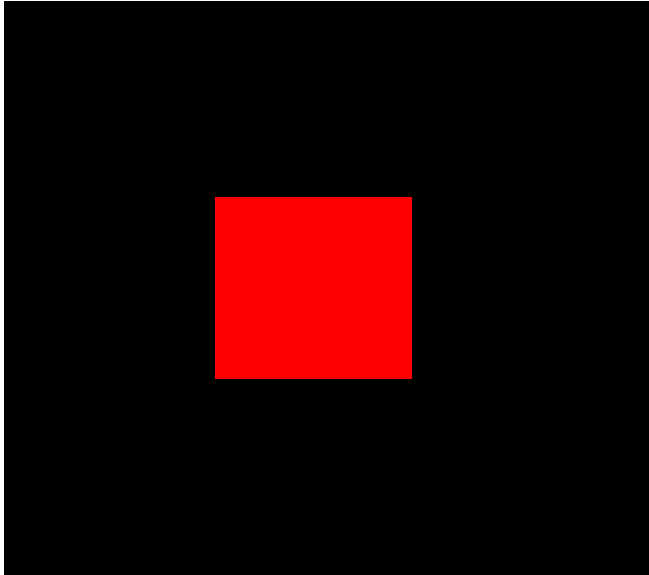


Convolution
mask



... holds only for periodic convolution

Discrete convolution via FT



Zero padding to the same size $(M+N-1)$

DFT calculation of both

Multiplication of the spectra

Inverse DFT

DFT calculation

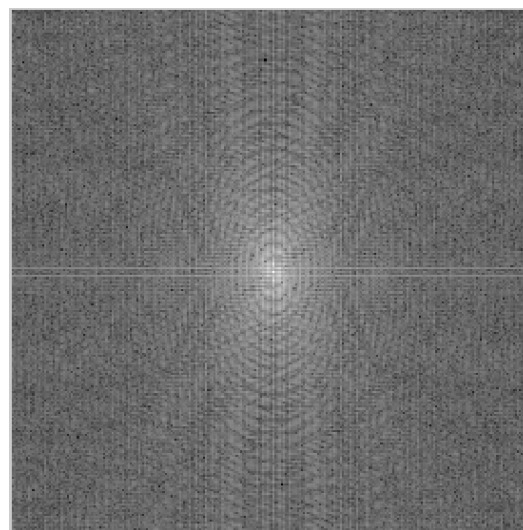
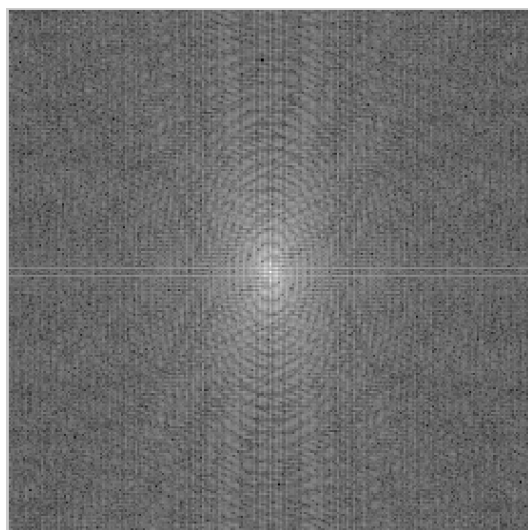
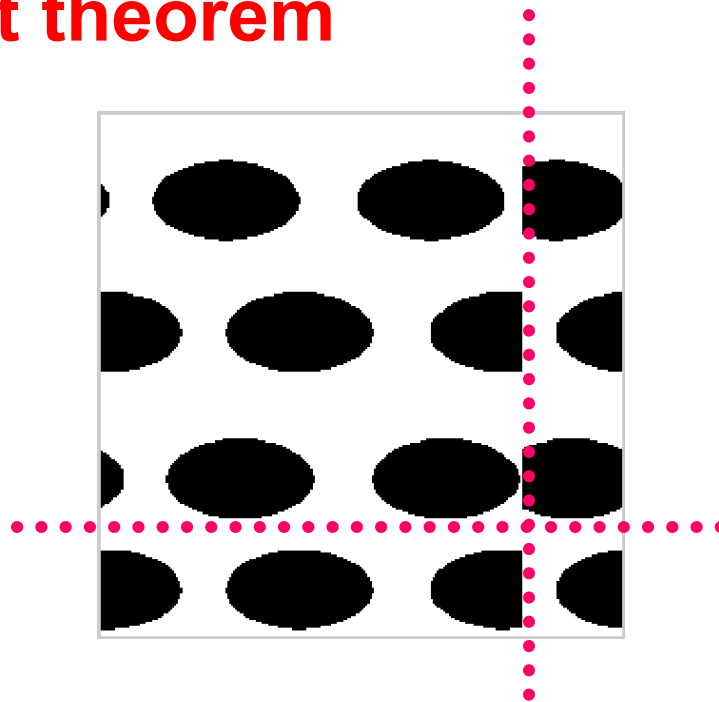
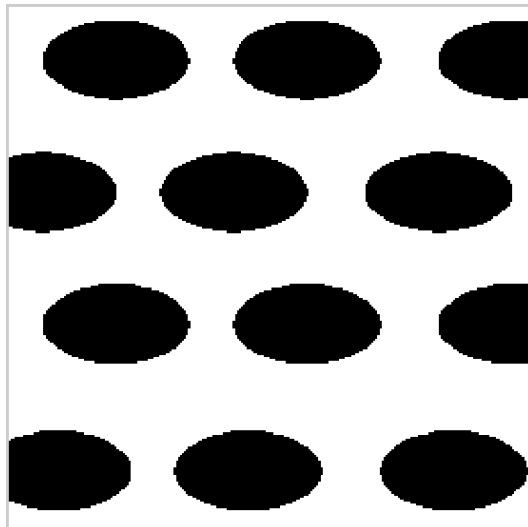
$$F_n \equiv \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N}$$

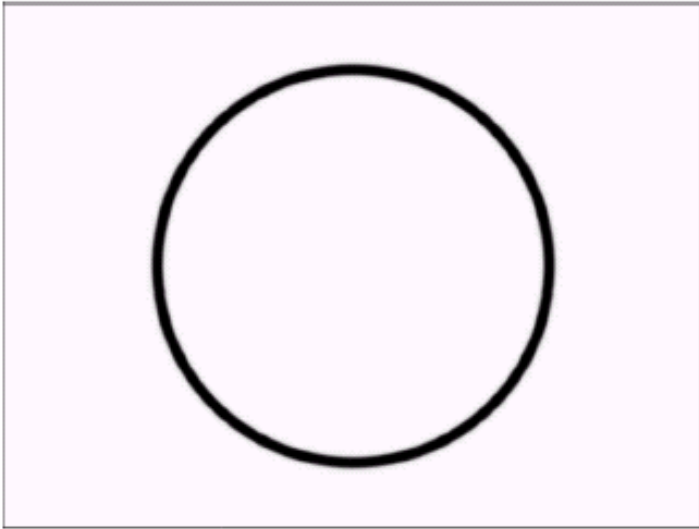
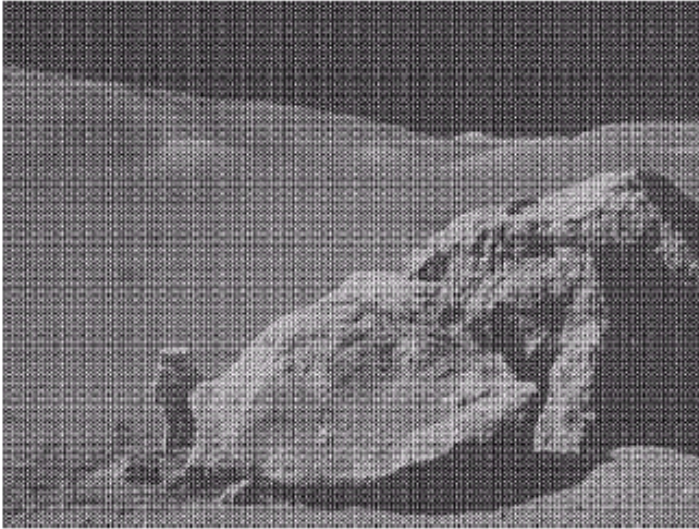
Directly - $O(N^2)$

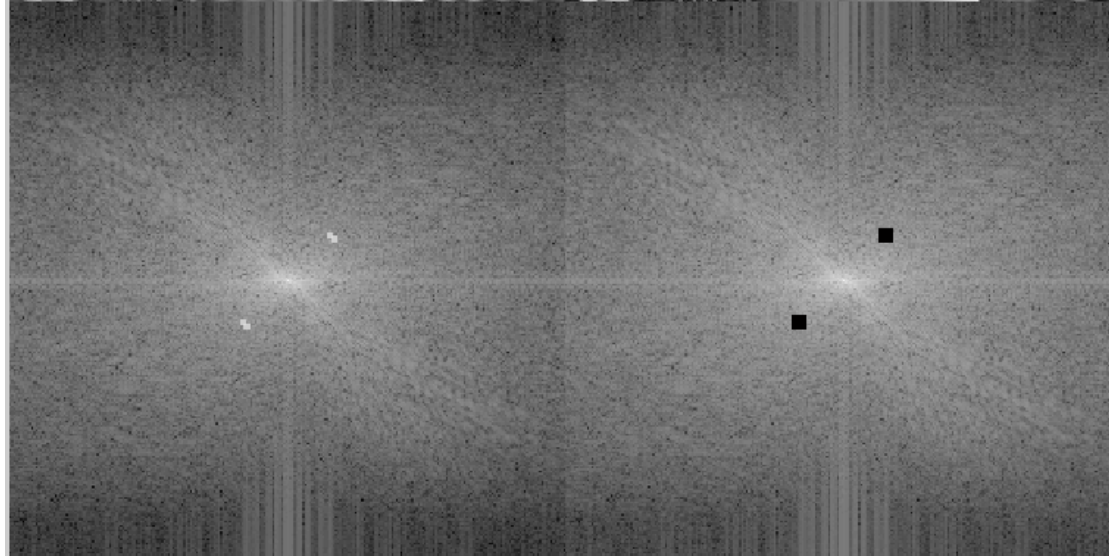
FFT - $O(N \log N)$ (Cooley, Tookey, 1960)

Compare to the complexity of convolution.

Discrete shift theorem





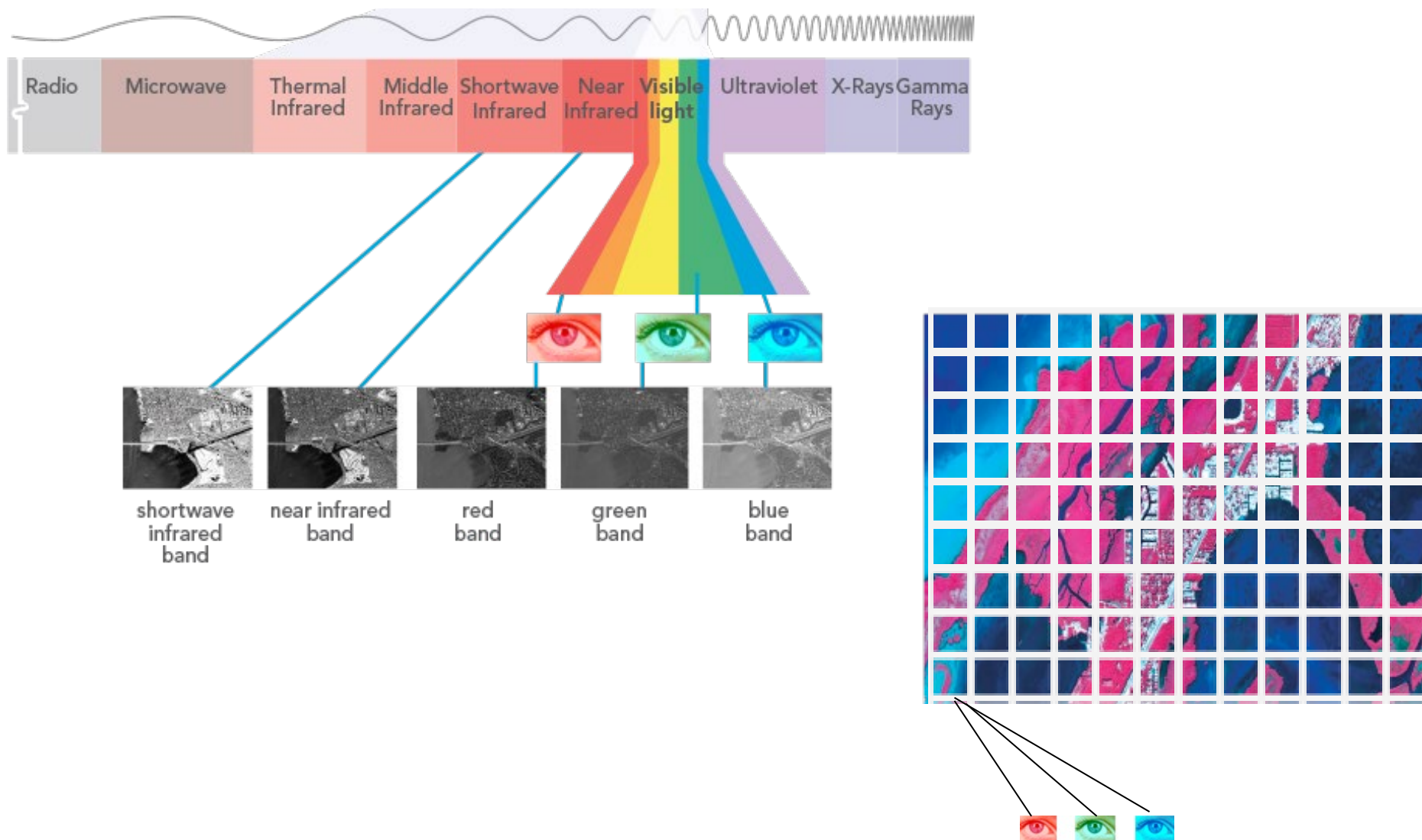


Filtrace periodického poškození



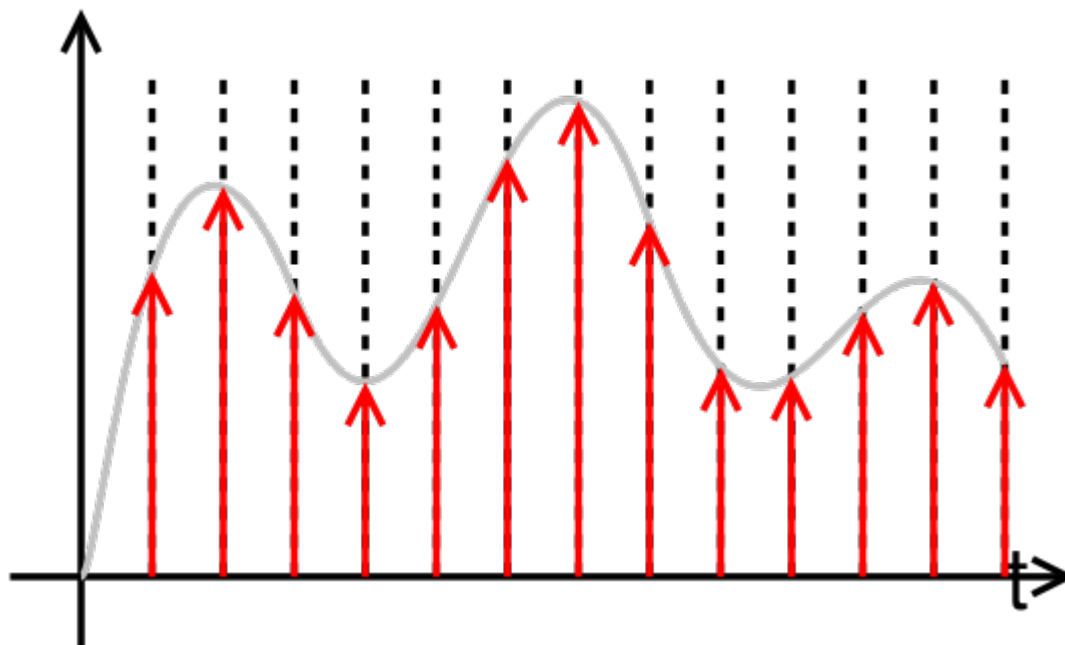


Digitalizace spojitého obrazu



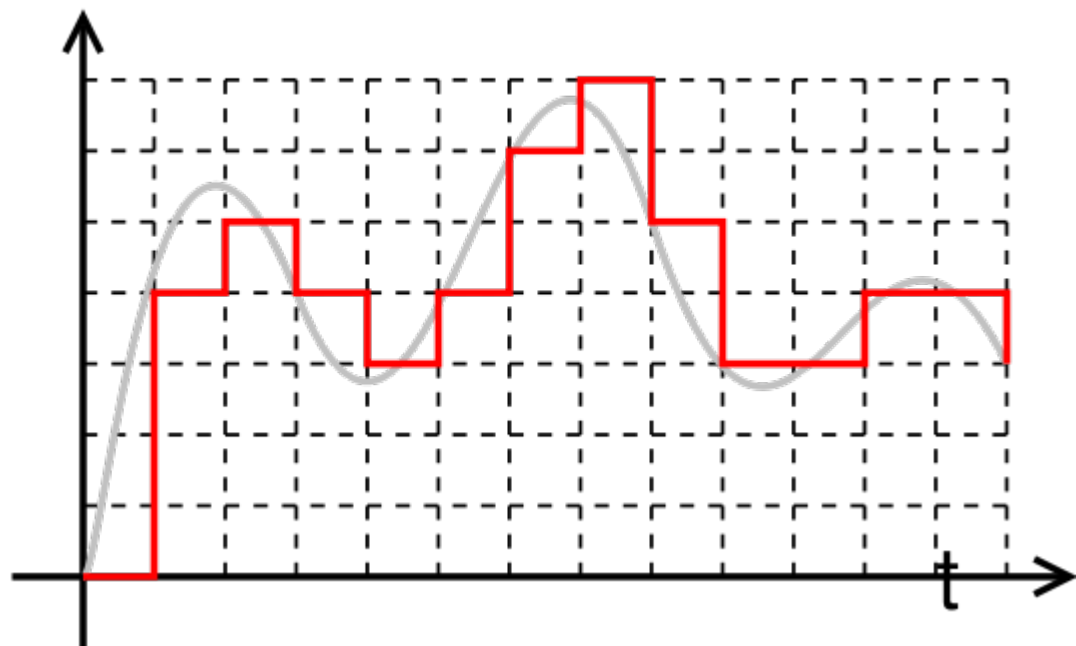
Digitalizace spojitého obrazu

- Vzorkování (sampling)



Digitalizace spojitého obrazu

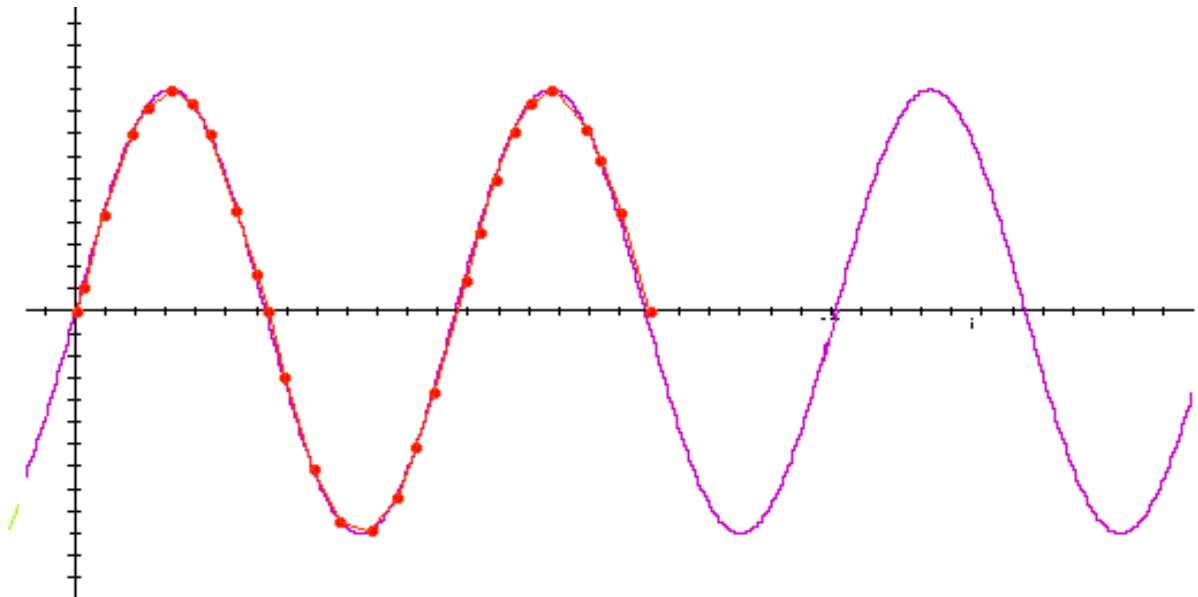
- Vzorkování (sampling)
- Kvantování



Vzorkovací teorém

Nyquist (1915), Kotelnikov (1933), Shannon (1945)

Lze původní obraz rekonstruovat?



Někdy ano, někdy ne. Kdy ano?

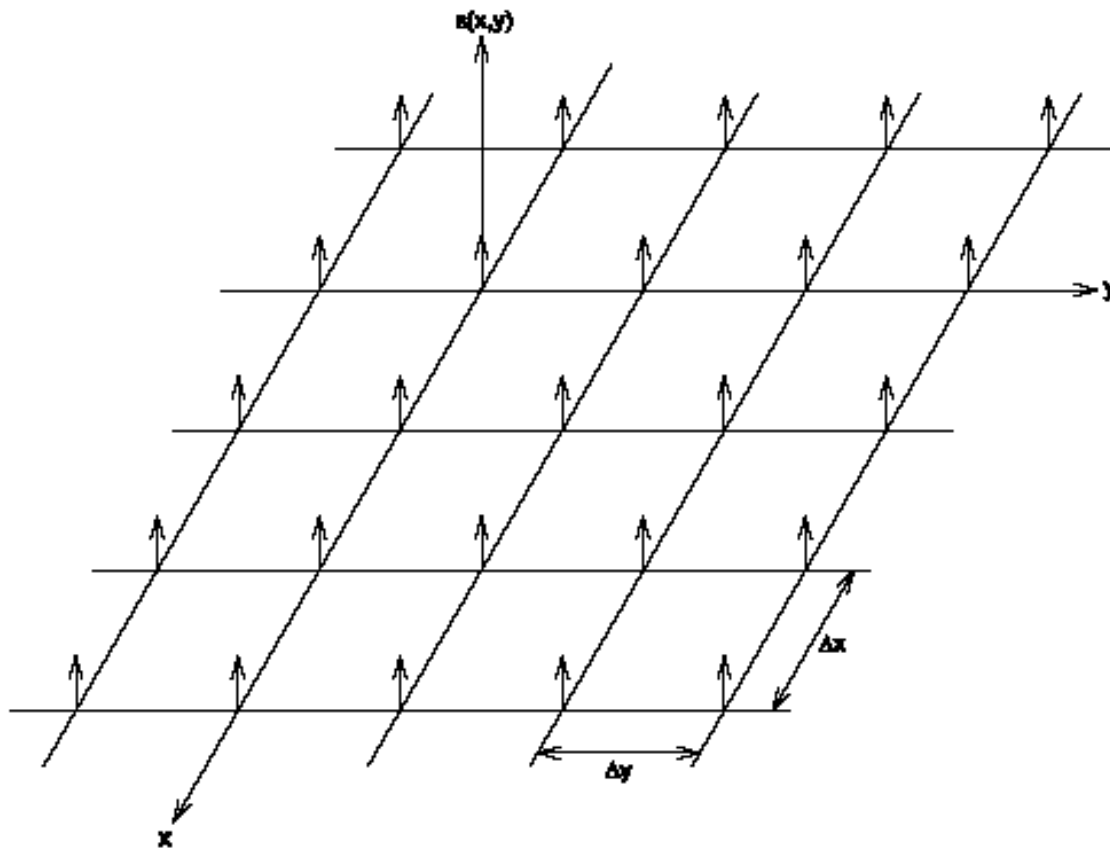
Matematický model vzorkování

Obrazová oblast

$$f(x, y) \cdot s(x, y) = d(x, y)$$

$$s(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta(x - i\Delta x, y - j\Delta y)$$

$$s(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta(x - i\Delta x, y - j\Delta y)$$



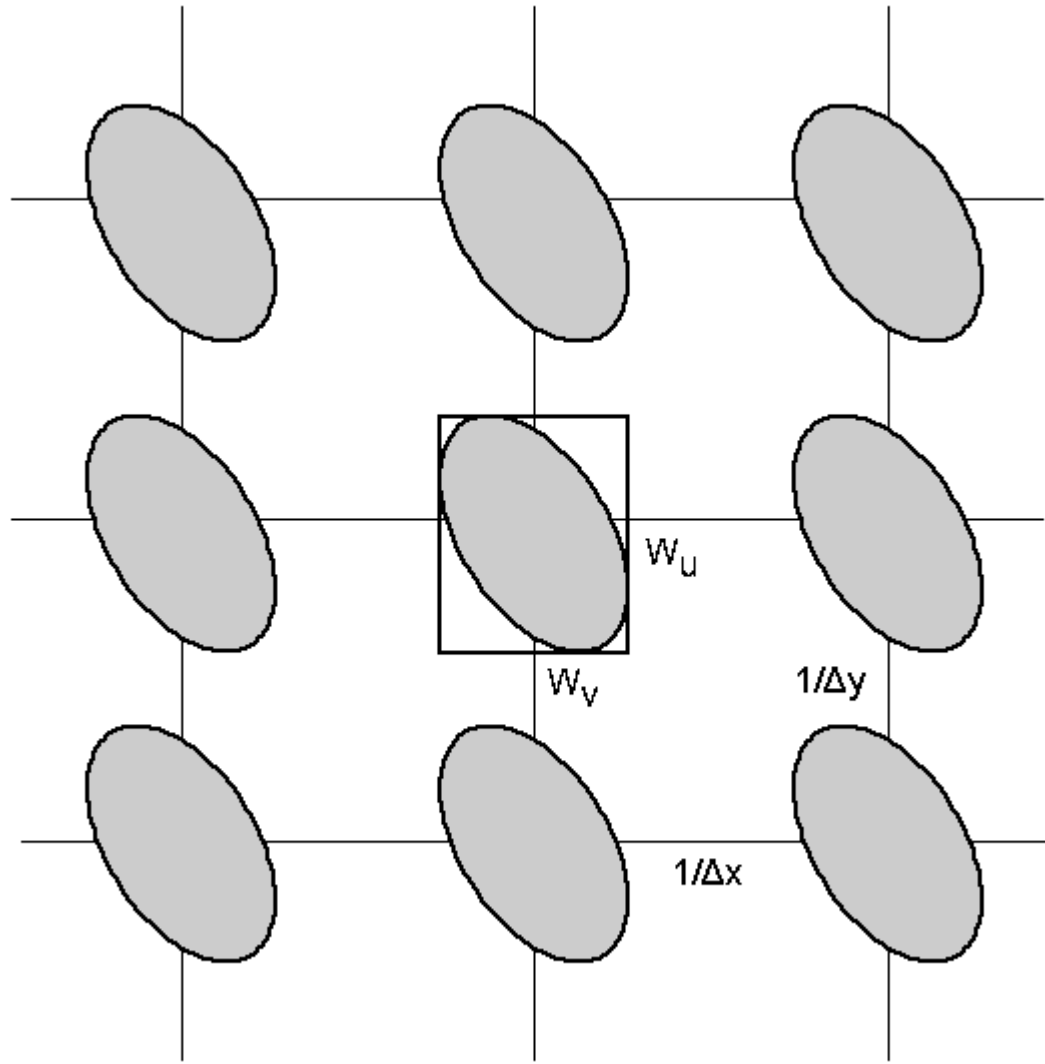
Matematický model vzorkování

Frekvenční oblast

$$D(u, v) = F(u, v) * S(u, v)$$

$$S(u, v) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta \left(u - i \frac{1}{\Delta x}, v - j \frac{1}{\Delta y} \right)$$

$D(u,v)$



Nyquistova podmínka

Vzorkování bez ztráty informace

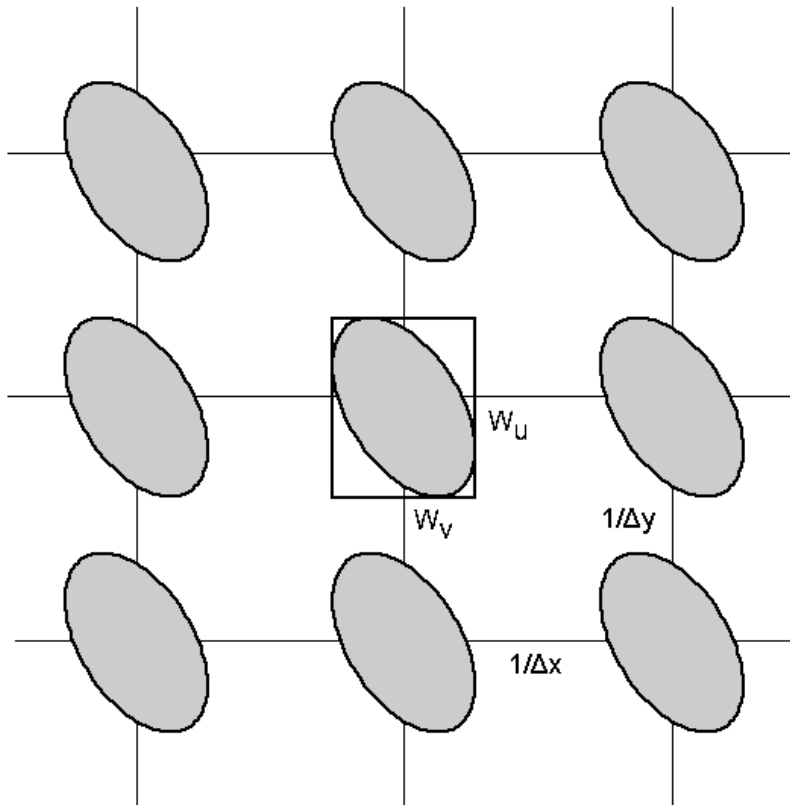
$$\Delta x \leq \frac{1}{2W_u}$$

$$\Delta y \leq \frac{1}{2W_v}$$

Zpětná rekonstrukce obrazu

Vyříznutí jednoho spektra a následná inverzní FT

Odpovídá interpolaci v obrazové oblasti

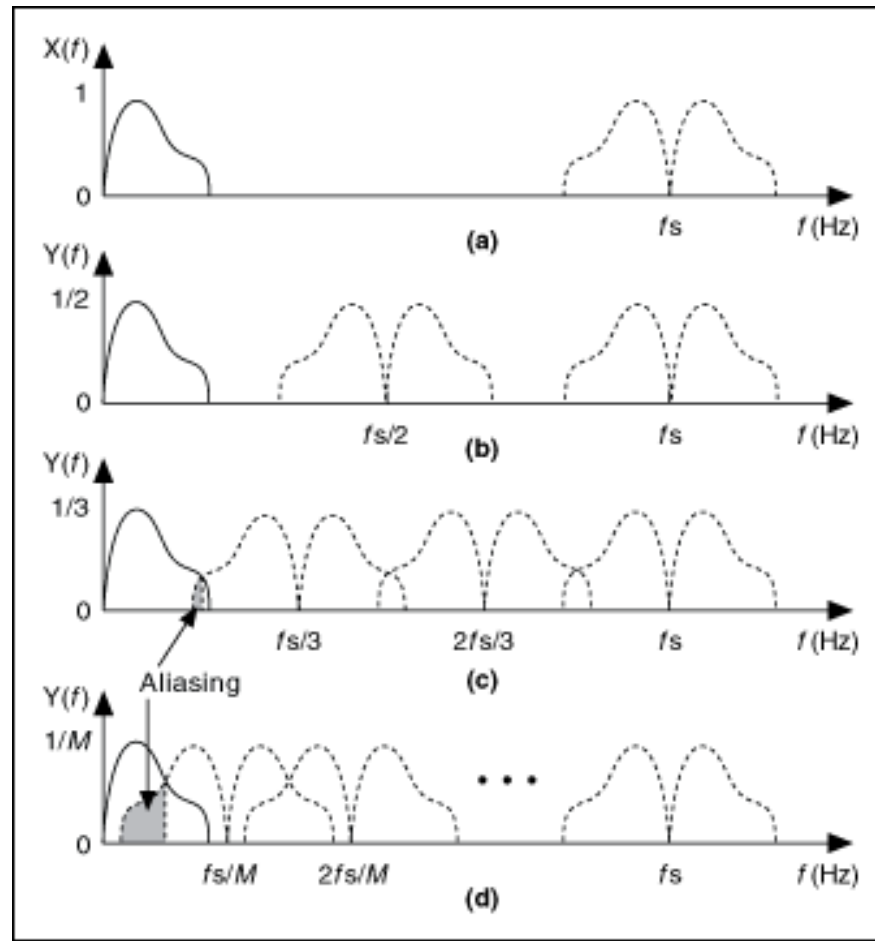


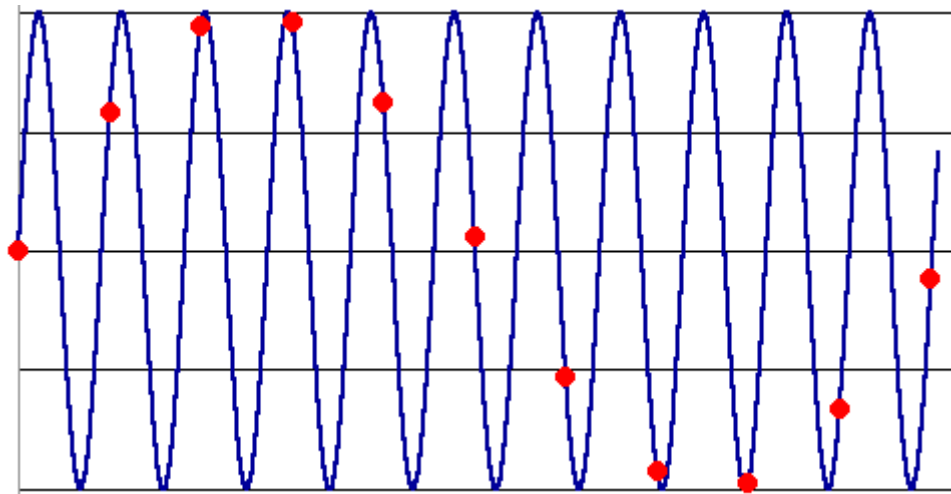
Vzorkování s nedostatečnou frekvencí

Moiré efekt – falešné nízké frekvence



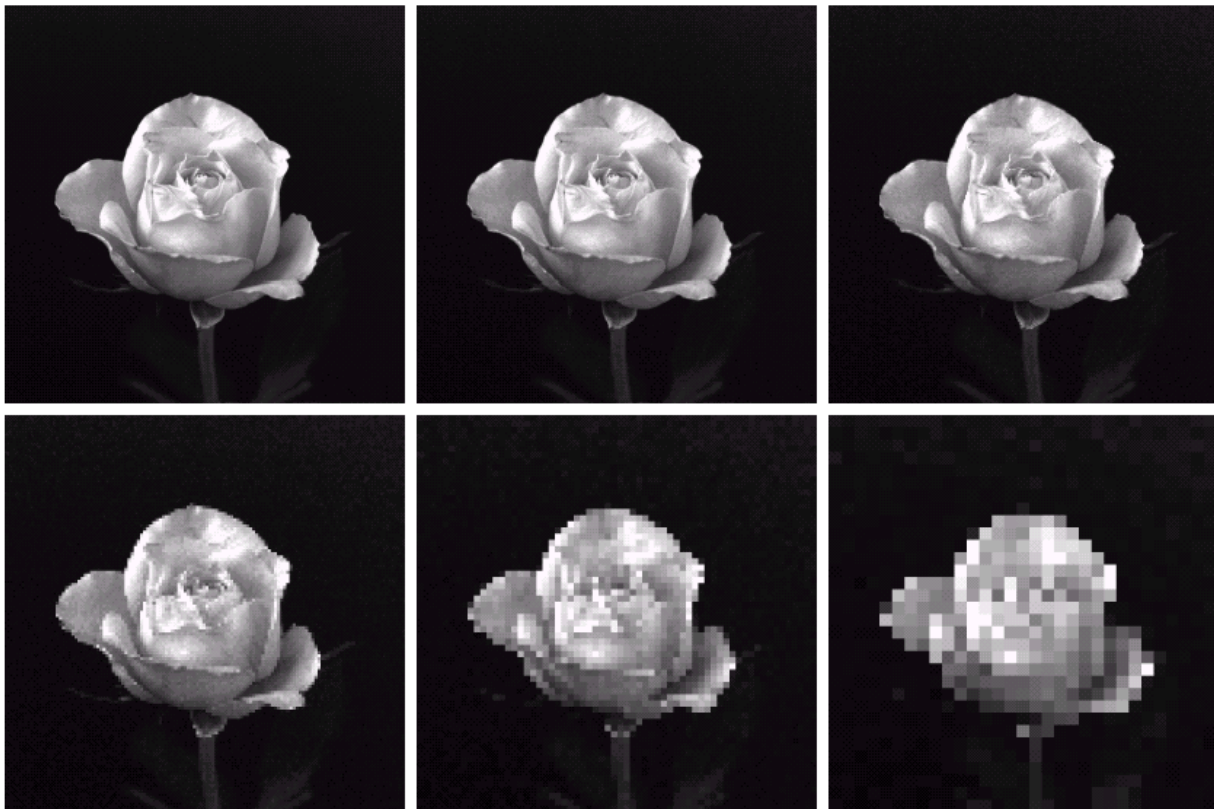
Vzorkování s nedostatečnou frekvencí





Vzorkování s nedostatečnou frekvencí

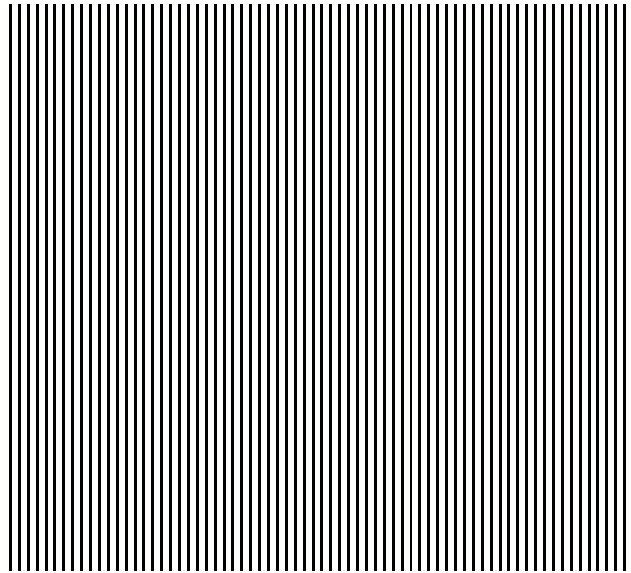
Překrytí sousedních spekter $D(u,v)$ □ ztráta VF informace (hrany, detaily, ...), aliasing



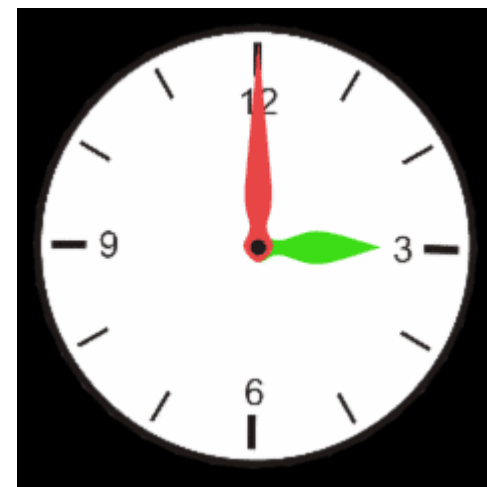
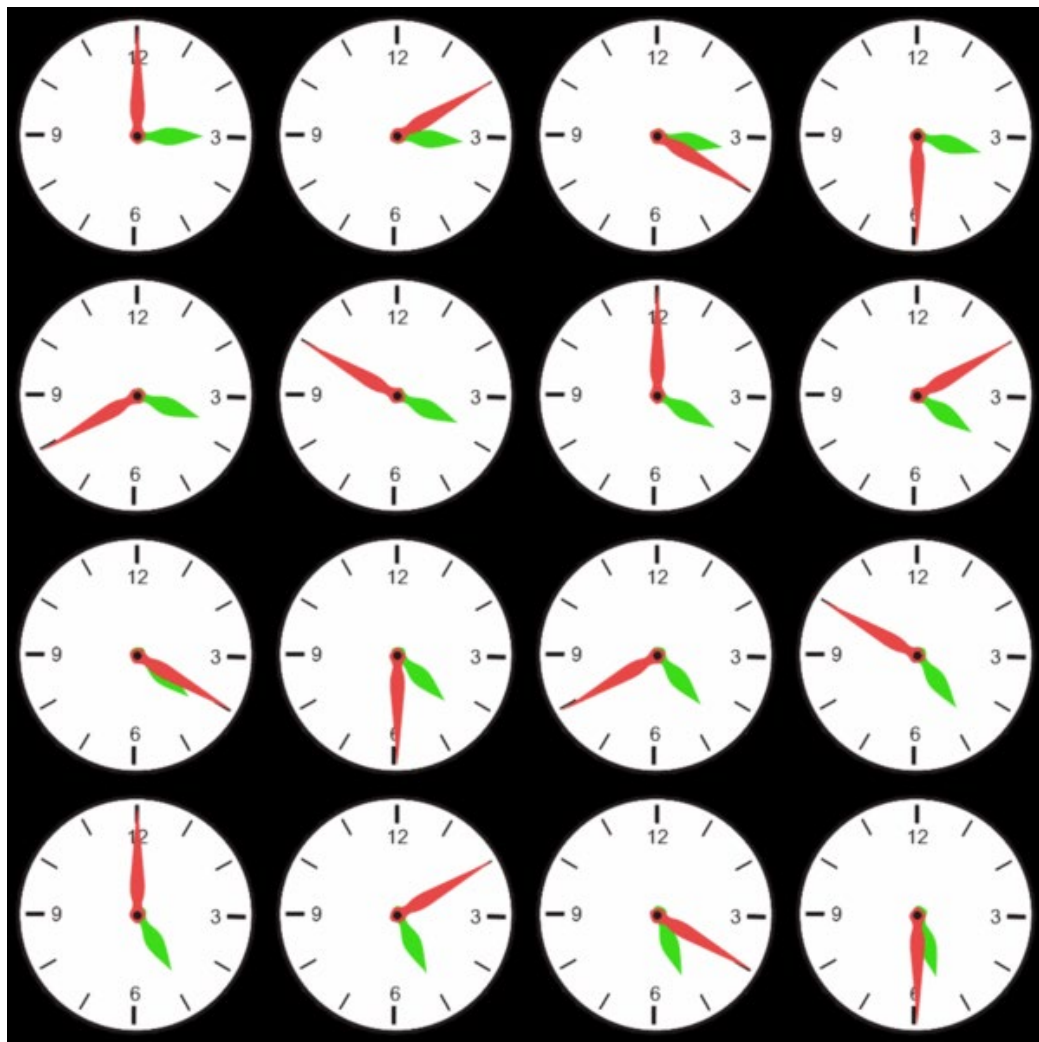


Vzorkování s nedostatečnou frekvencí

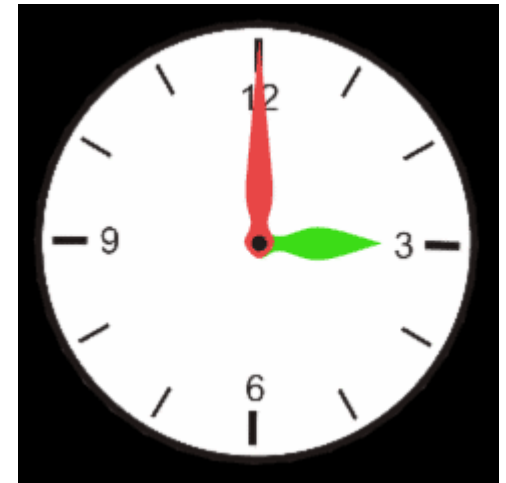
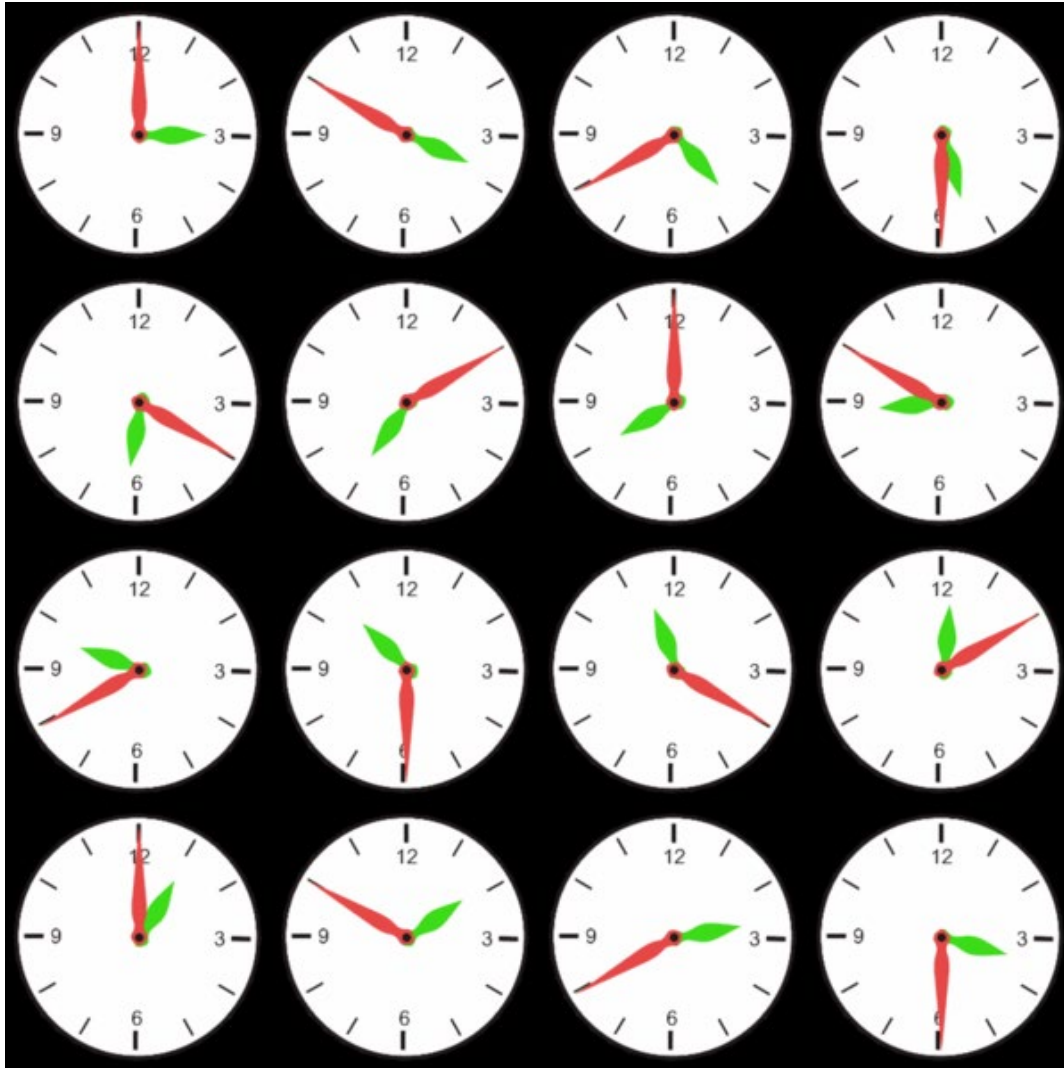
Moiré efekt – falešné nízké frekvence



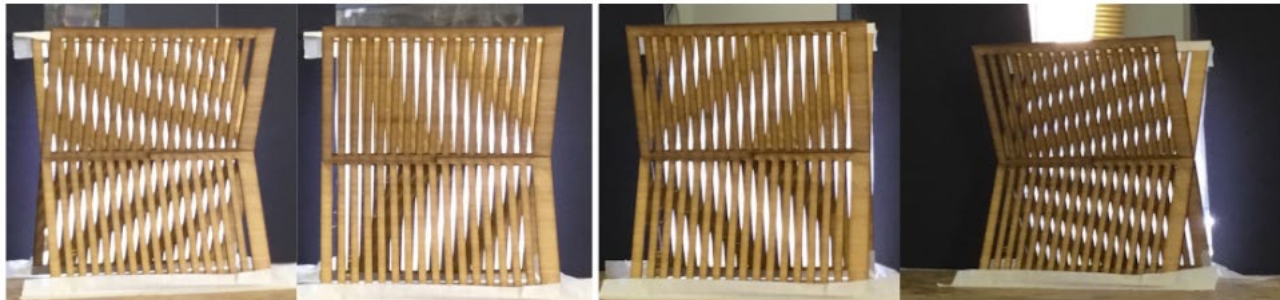
Moiré efekt v časové oblasti



Moiré efekt v časové oblasti

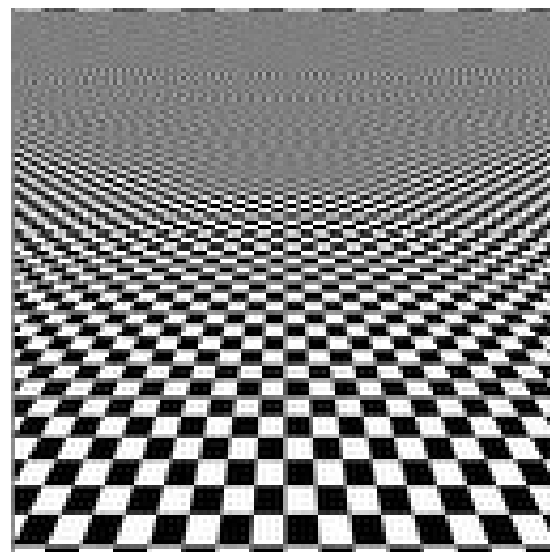
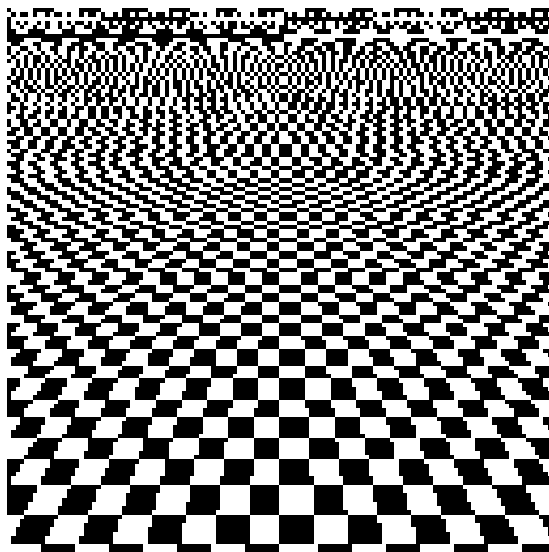


Moiré efekt užitečný – námořní navigace



Anti-aliasing techniky

- Zvýšení vzorkovací frekvence
- Odstranění vysokých frekvencí před vzorkováním



Vzorkování v reálných optických systémech

Rastr je omezený

Jen několik možných vzorkovacích frekvencí

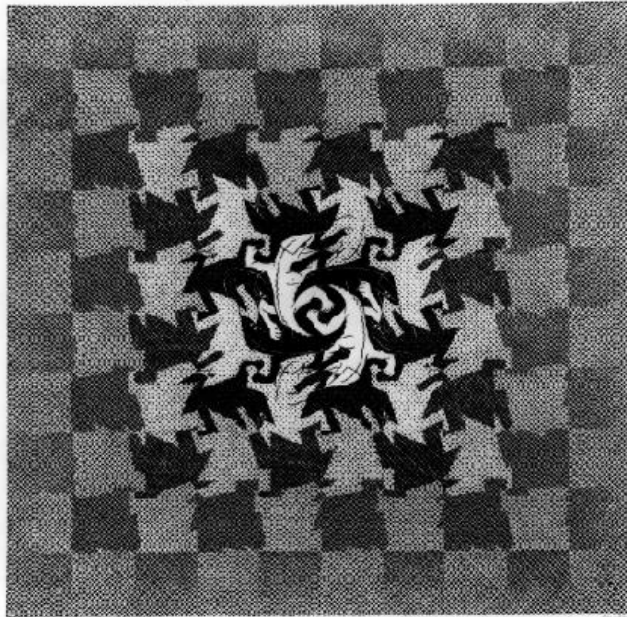
Vzorkování není pomocí δ – funkcí

Optika působí jako low-pass filtr

Netradiční vzorkování

Nepravoúhlý rastr (rovnoběžník, hexagon, ...)

- co nejlépe pokrýt rovinu (u,v) pomocí $\text{supp}(F)$



Netradiční vzorkování

Nepravoúhlý rastr (rovnoběžník, hexagon, ...)

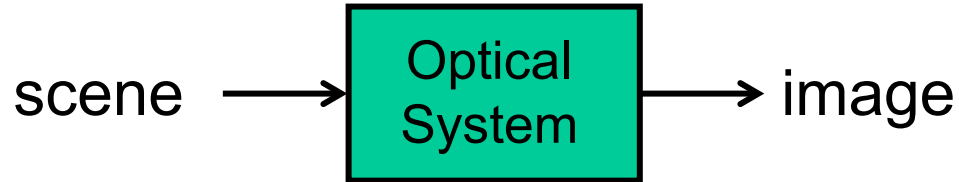
- co nejlépe pokrýt rovinu (u,v) pomocí $\text{supp}(F)$

Adaptivní vzorkování – proměnná frekvence dle charakteru obrazu

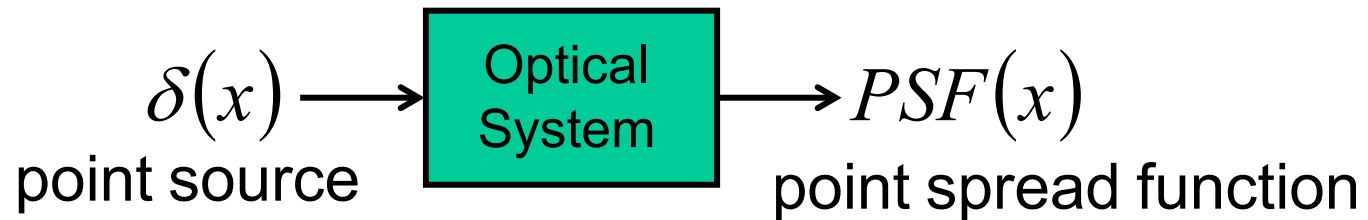
Compressive sensing

Běžné kamery a scannery neumožňují ani jedno

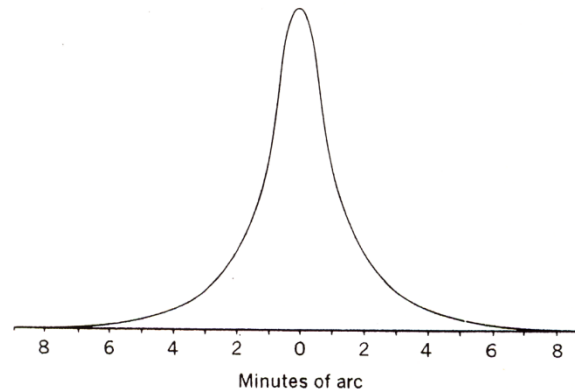
PSF - Point Spread Function



- teoreticky Dirac delta function.
- realita jiná



- PSF lidského oka



PSF - Point Spread Function



normální



krátkozrakost

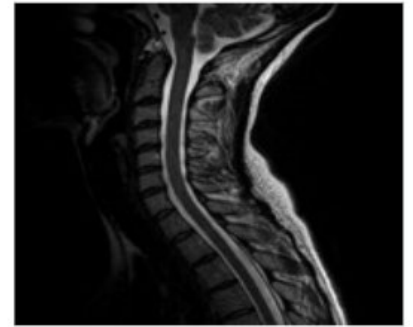
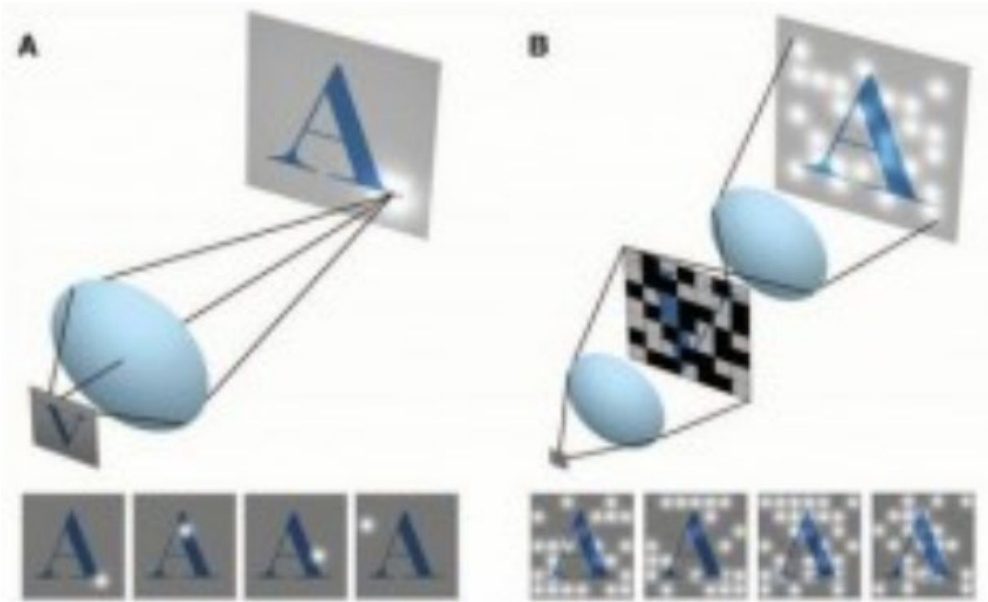


dalekozrakost

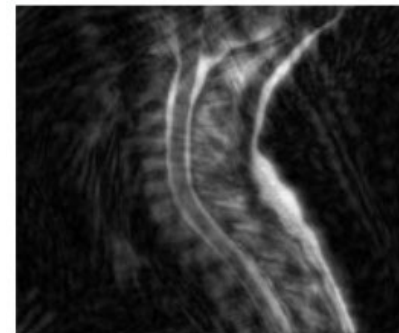


astigmatismus

Compressed sensing



C: Hewit J.



6X undersampled



6X undersampled with CS reconstruction

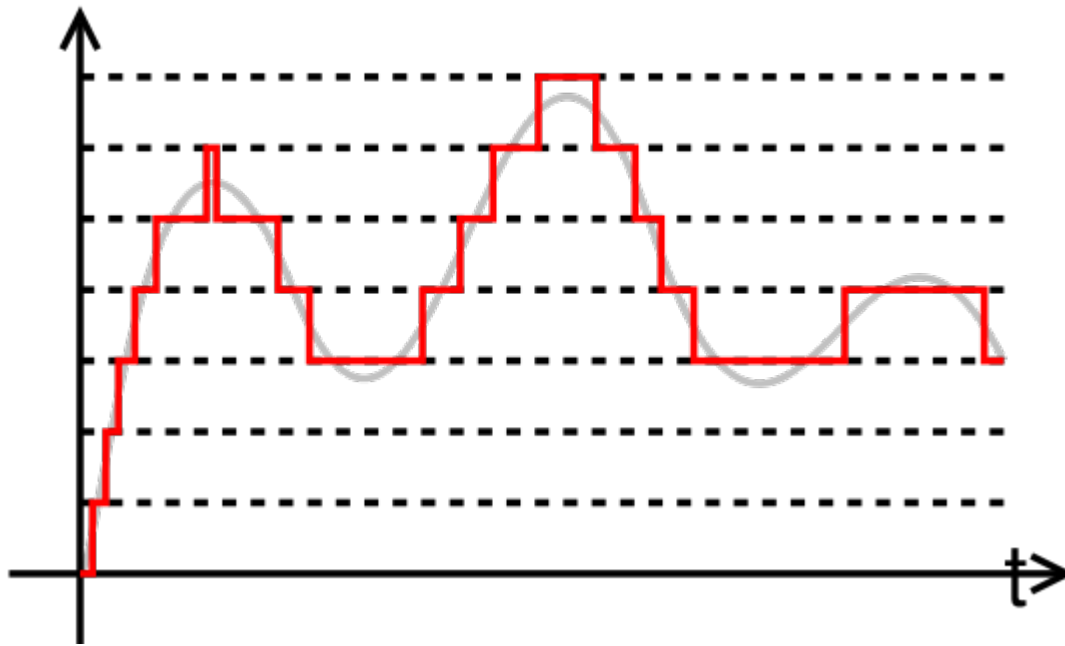
Kvantování obrazu

Kvantování – diskretizace oboru hodnot signálu
-- vždy ztrátové

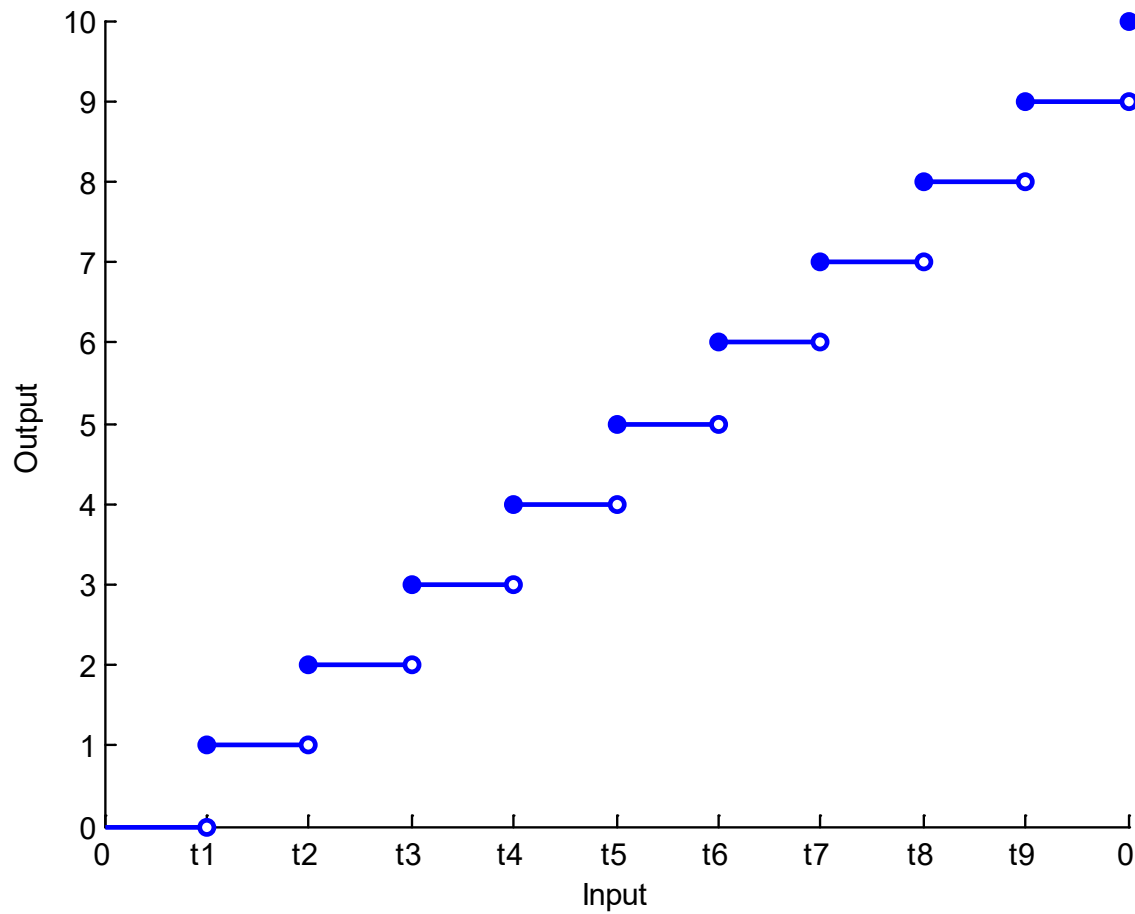
Kvantizér $Q: R \sim L$

$$L = \{0, 1, \dots, k\} \quad (k = 255)$$

Kvantovaný signál

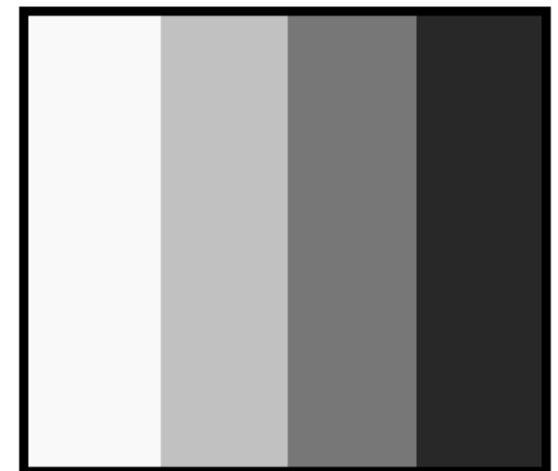
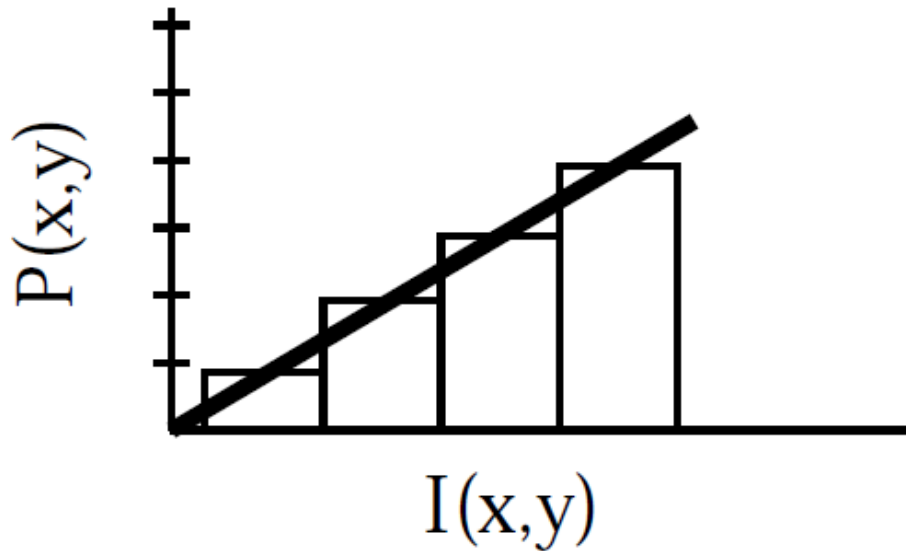


Kvantizér



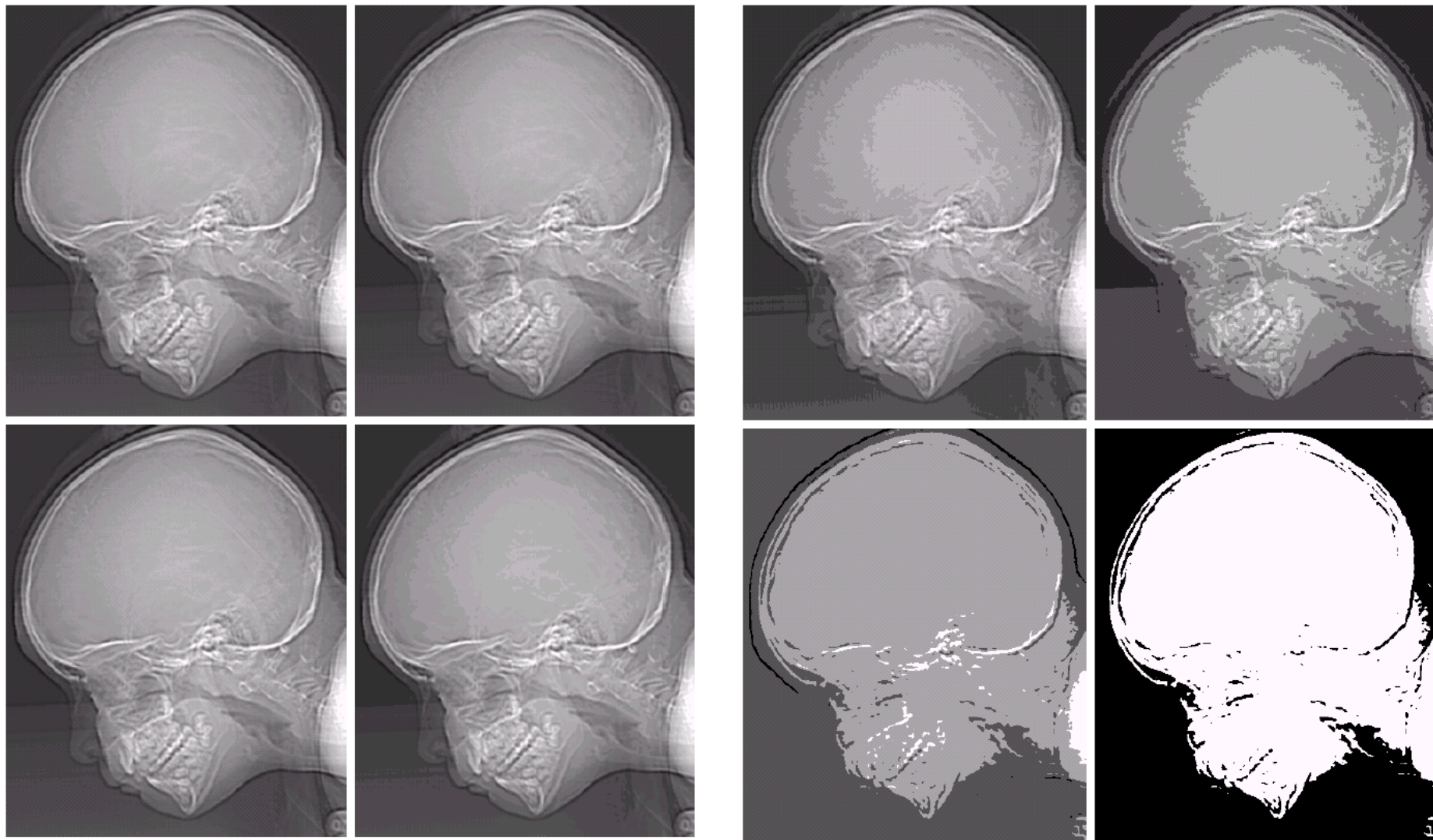
Kvantizér

$$P(x, y) = \text{trunc}(I(x, y) + 0.5)$$



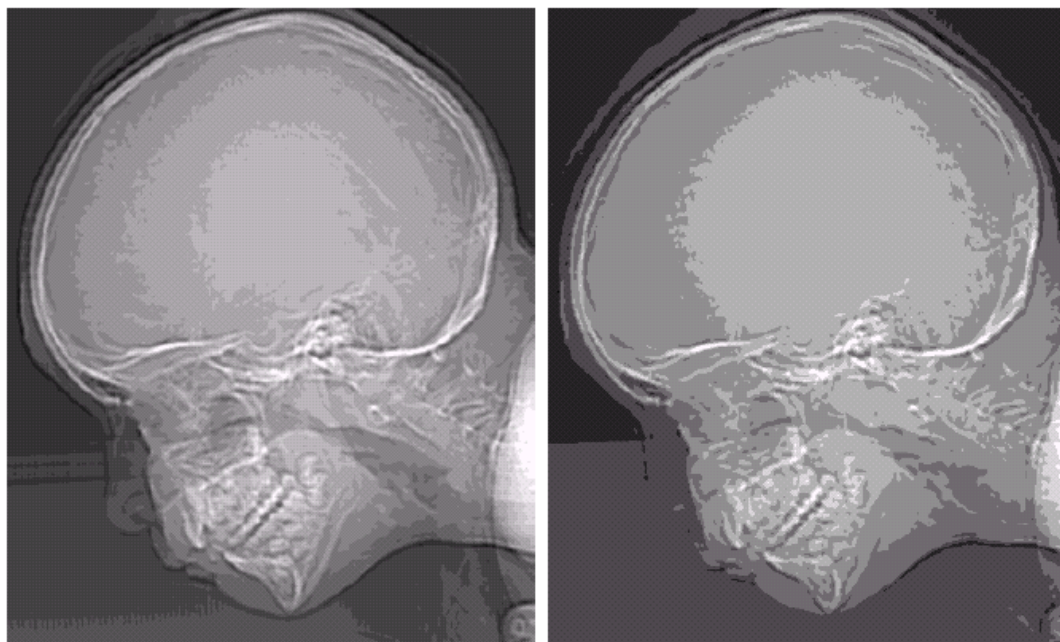
$P(x, y)$
(4 bits per pixel)

Kvantování do různého počtu úrovní



Vznik falešných kvantizačních hran

(Kvantizační šum)



Účinek vzorkování a kvantování na lidský zrak



8 bits / pixel



4 bits / pixel



2 bits / pixel

Účinek vzorkování a kvantování na lidský zrak

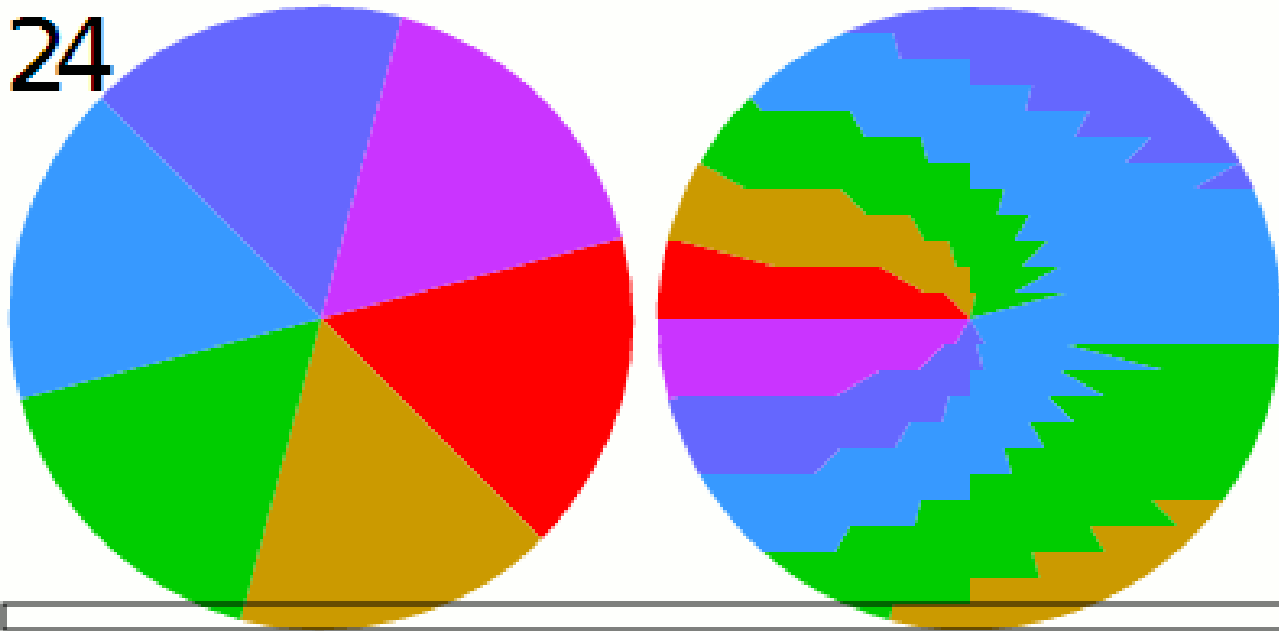




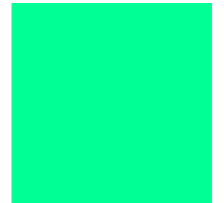
<http://scalarmotion.wordpress.com/2009/03/15/propeller-image-aliasing/>

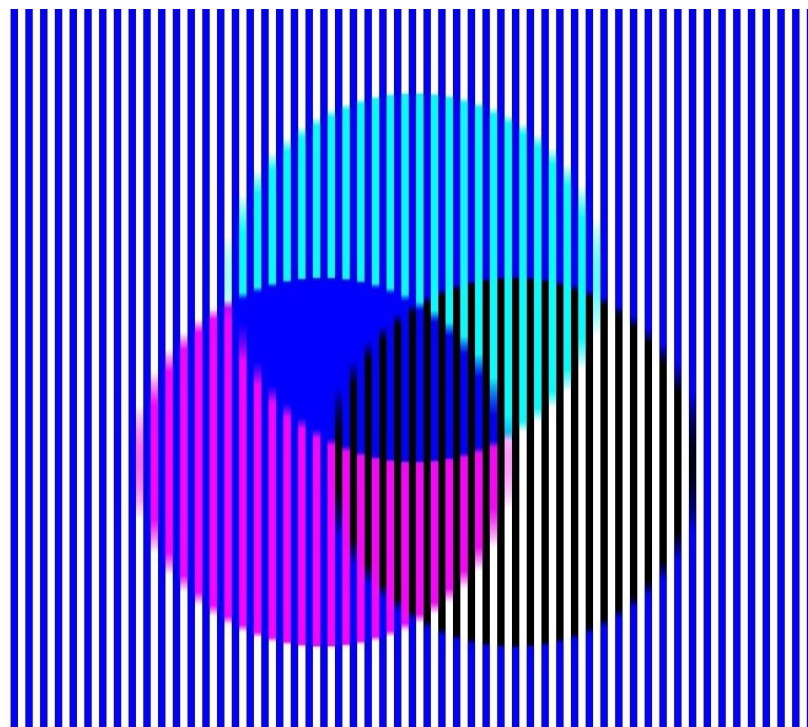
Rolling shutter

24



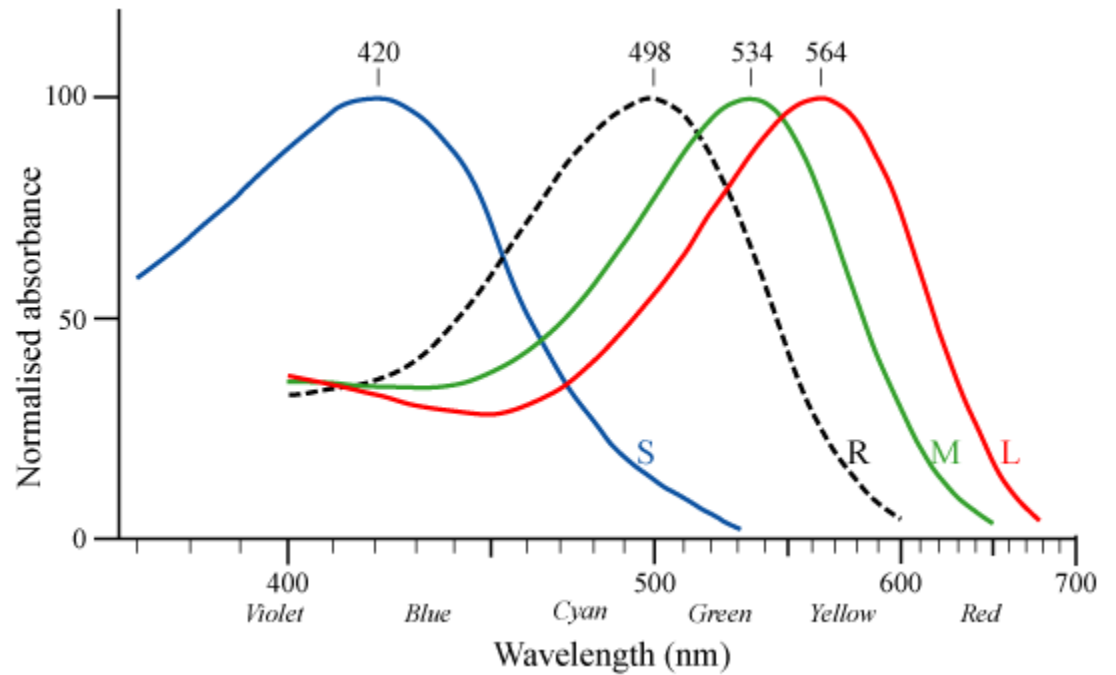
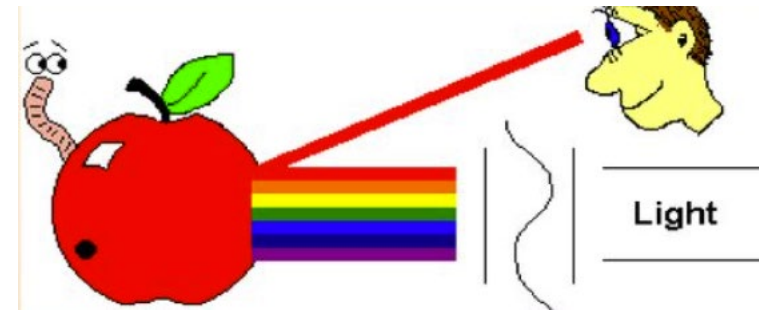
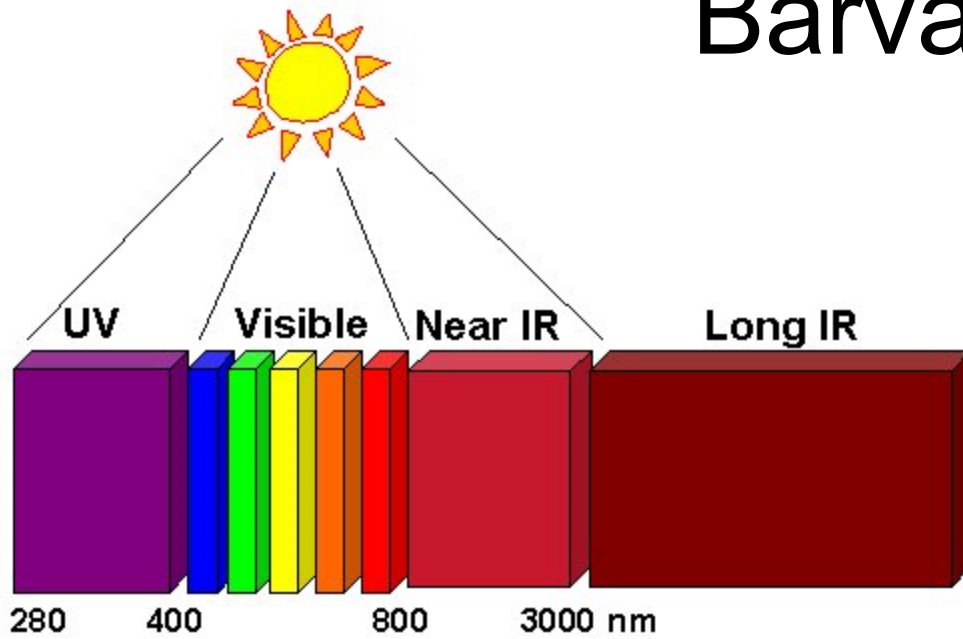
Barva



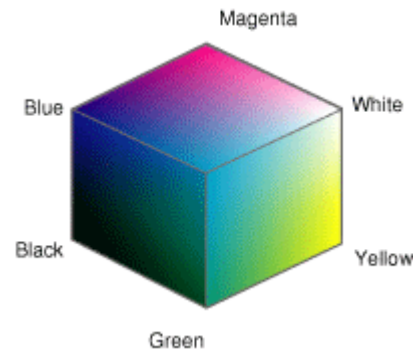




Barva



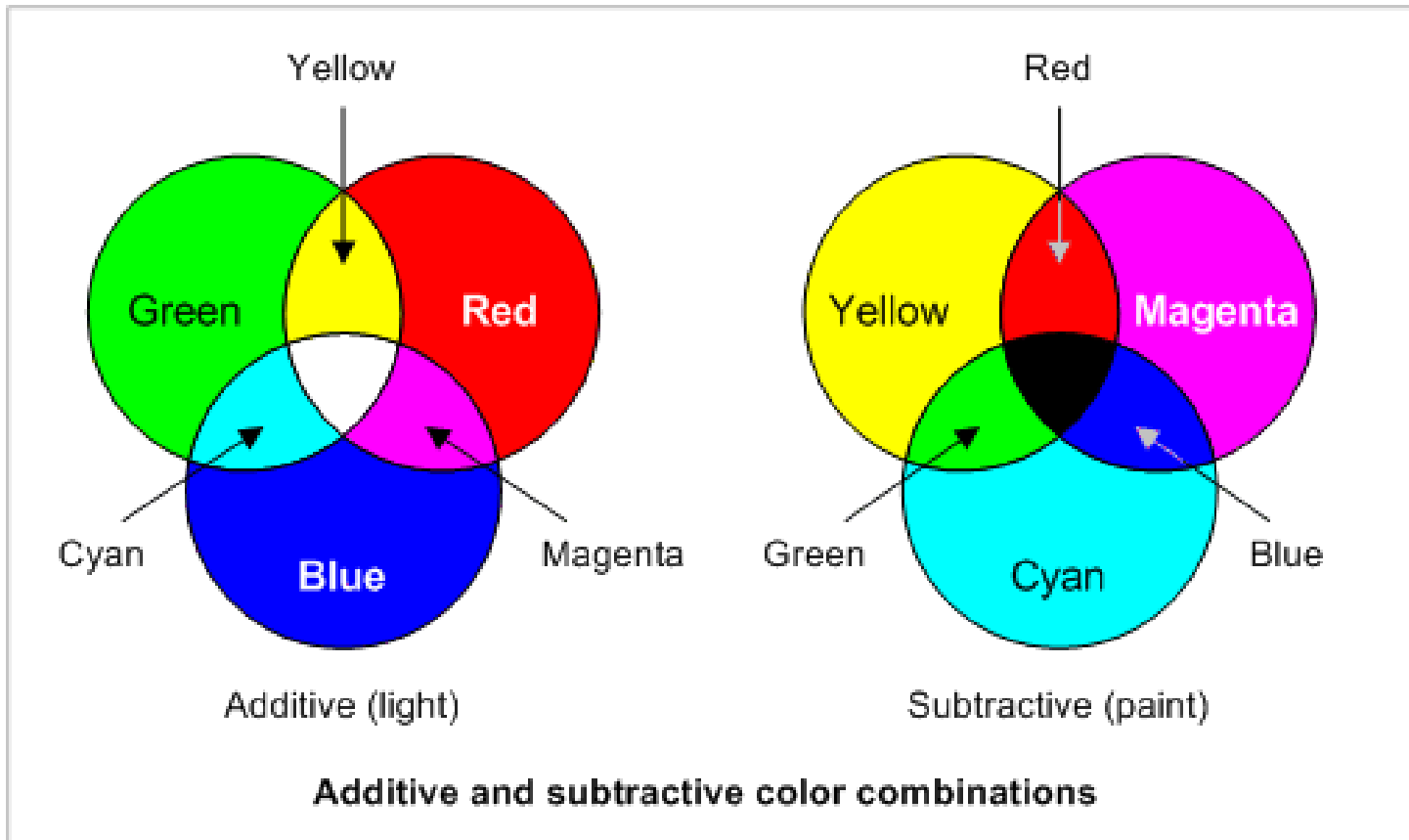
Barevné prostory



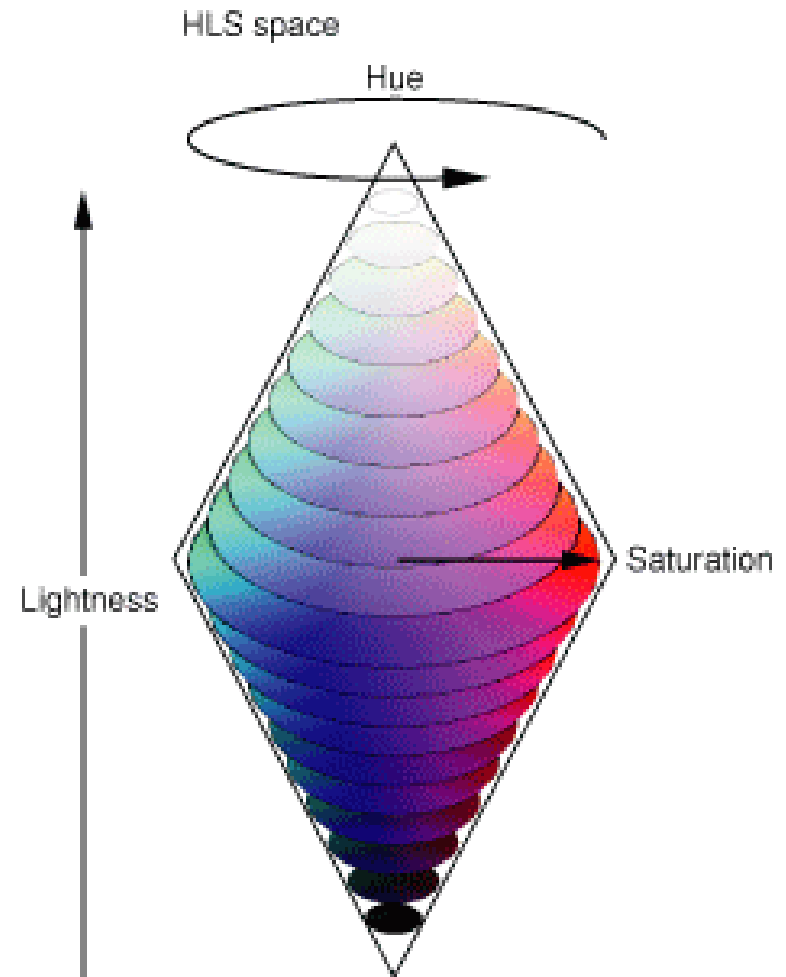
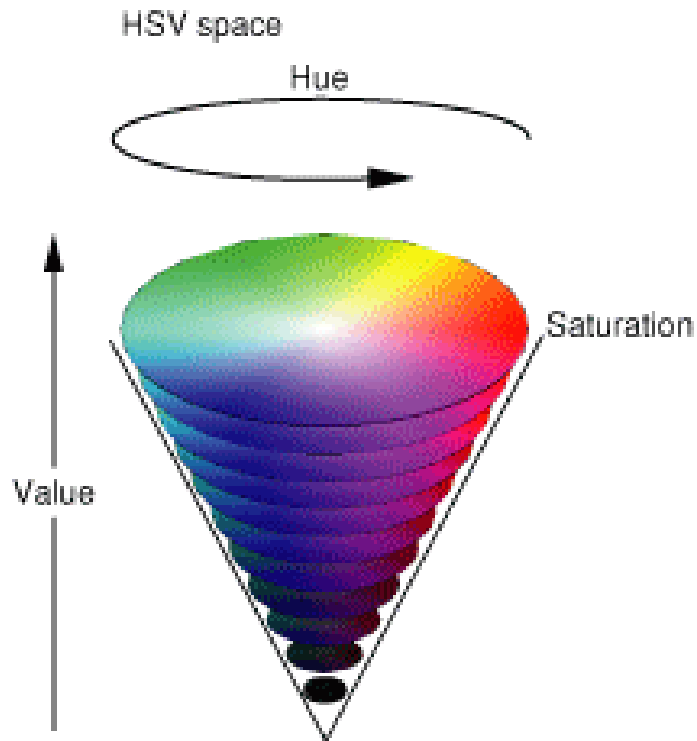
- RGB aditivní
- CMYK subtraktivní (tisk)
- „ostatní“ ... HSV, HSL,
YIQ, YUV (NTSC, PAL)

- na zařízení nezávislé prostory
(sRGB ... CRT)
- profile connection space (PCS)
CIE XYZ nebo CIELAB

Barevné prostory



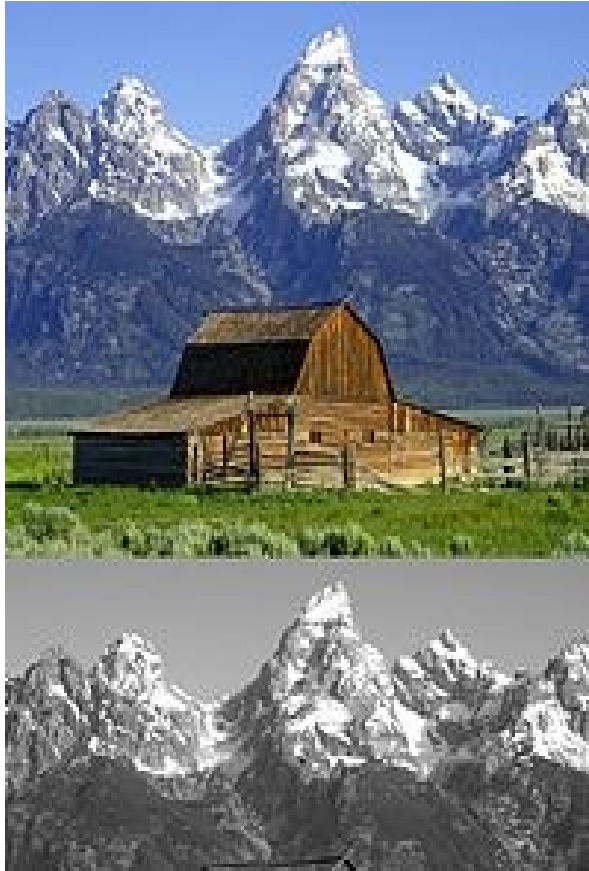
Barevné prostory



HSV ... hue, saturation, value

HLS ... hue, lightness, saturation.

Barevné prostory

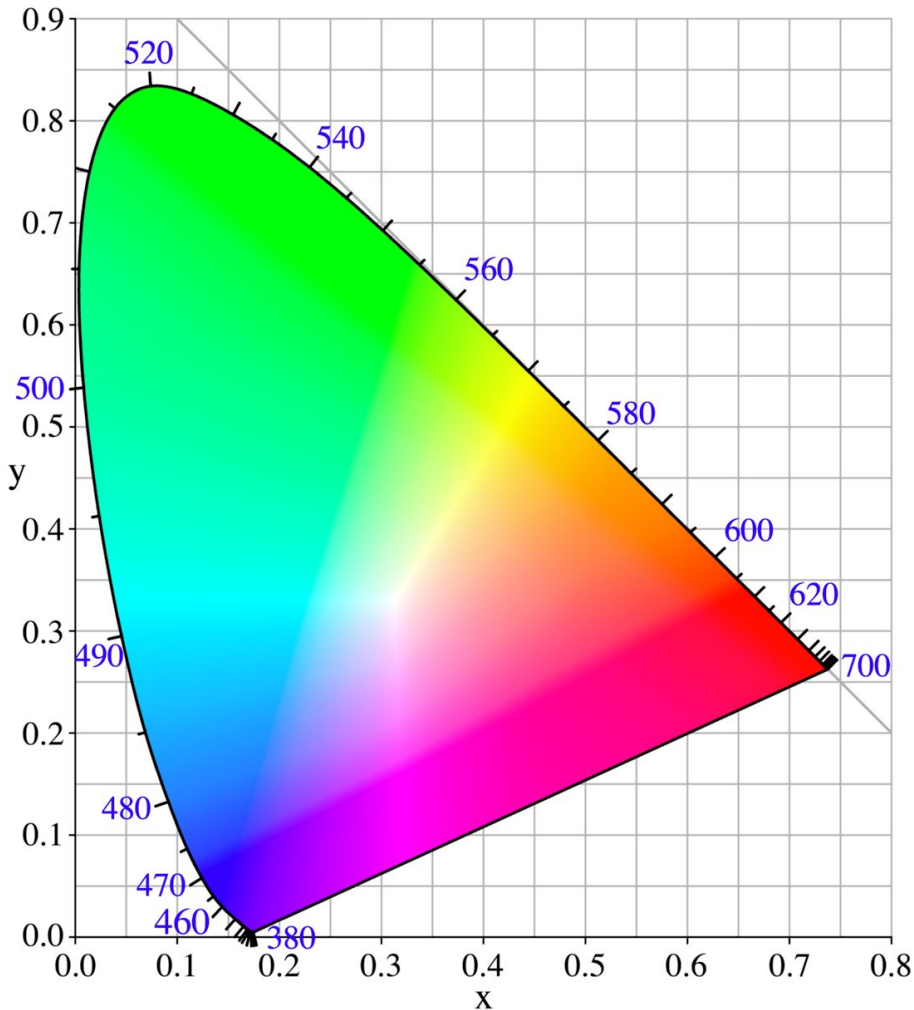


YIQ ...
NTSC

Y intenzita, I, Q barevnost

CIE XYZ

CIE 1931 XYZ



ICC standard

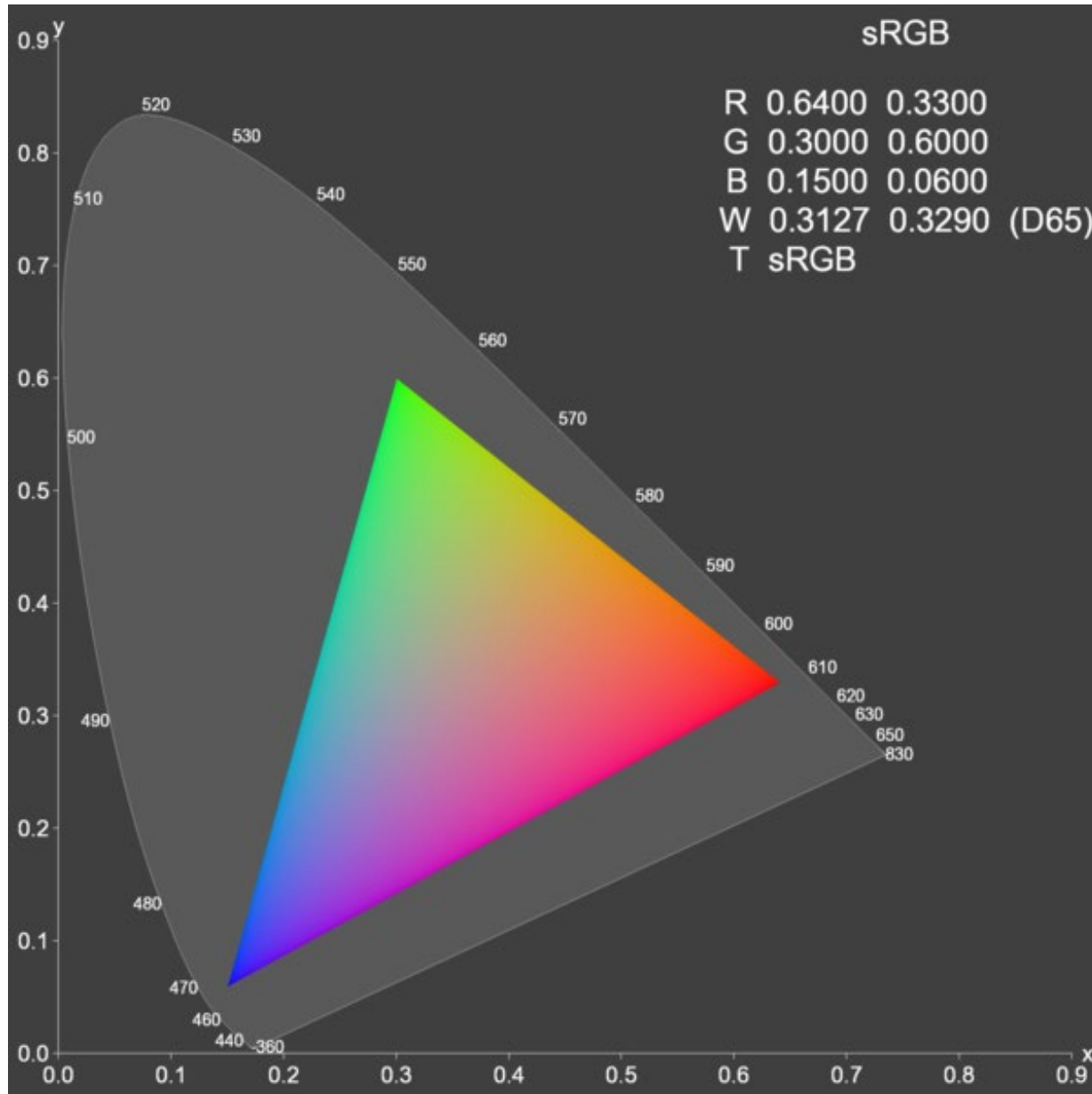
předloha odrážející světlo
nasvícené pod úhlem 45 stupňů
pozorovaném ve směru kolmém
při osvětlení o síle 500 luxů
standardní iluminant D509
(denní světlo

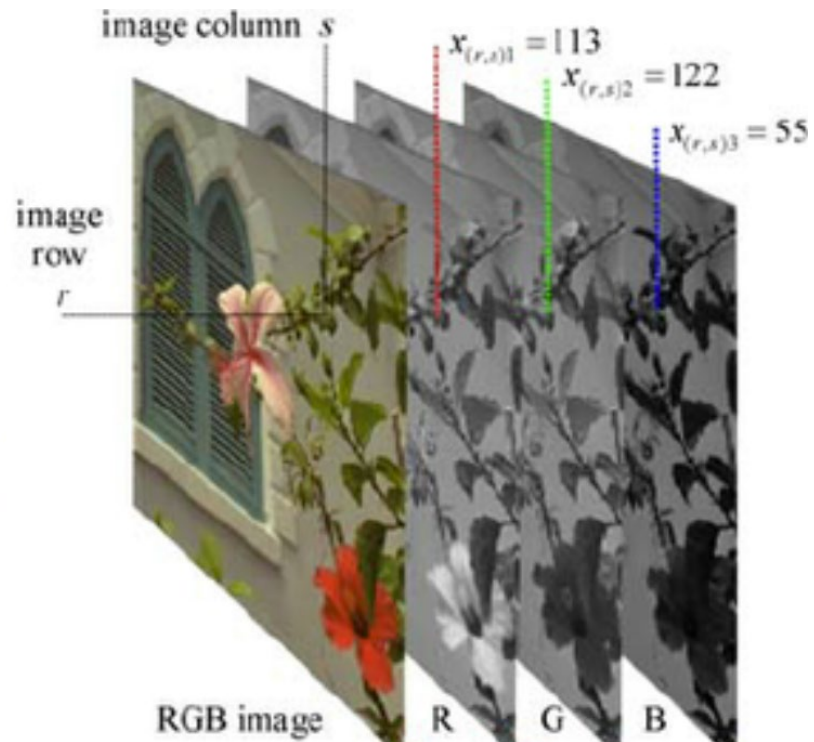
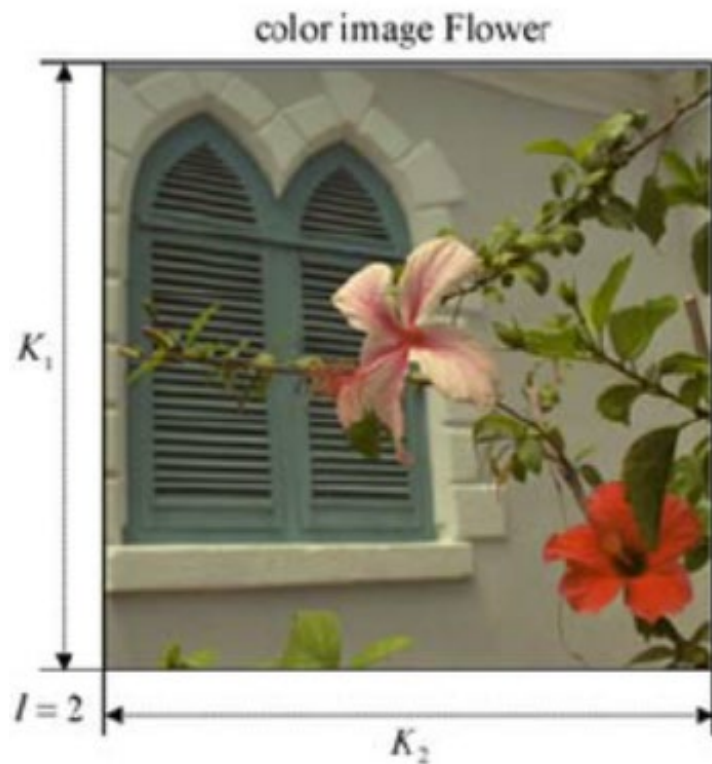
o barevné teplotě 5000 K)

1920s,

W. D. Wright and John
Guild

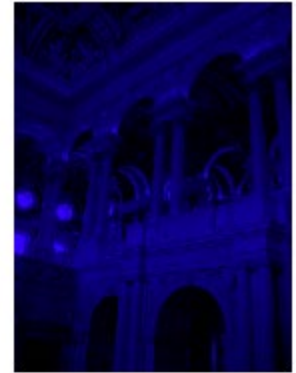
Barva - gamut





R	255	255	255	125	0	0	0	0	0	125	255	255
G	0	125	255	255	255	255	255	125	0	0	0	0
B	0	0	0	0	0	125	255	255	255	255	255	125

RGB (K. Plataniotis)



RGB



HSV



YUV